



Bi-level Optimization of Residential Community Energy Systems: Space Heating and DHW Loads under Multi-Energy Pricing

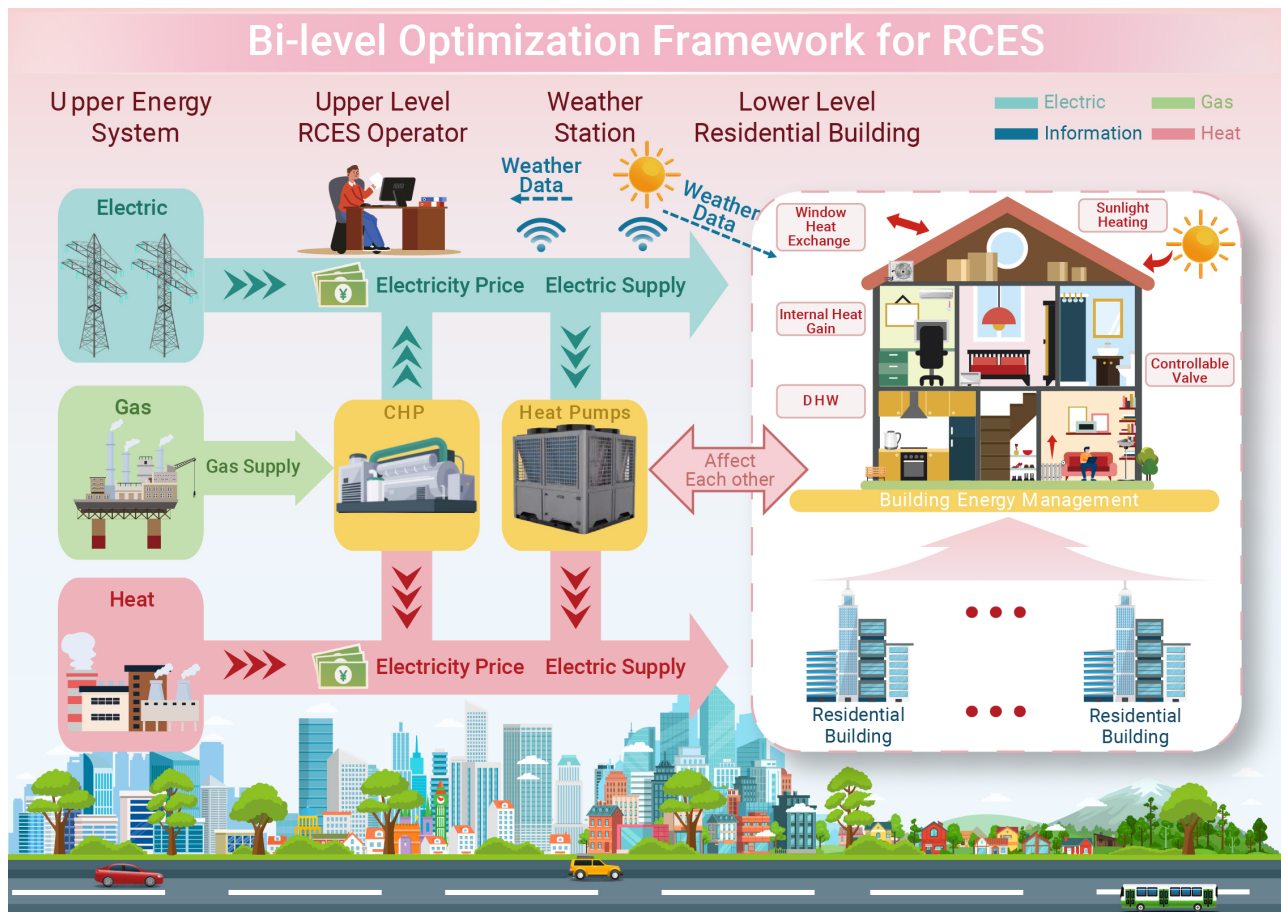
Shuo Liang,^{1,2} Xiaolong Jin,^{1,2,*} Hongjie Jia,^{1,2} Yunfei Mu,^{1,2} Xiaodan Yu,^{1,2} Yuming Zhao,³ and Shan He³

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GRAPHICAL ABSTRACT



PUBLIC SUMMARY

- Developed a hierarchical optimization approach for RCES managing space heating and DHW under multi-energy pricing.
- Upper-level model optimizes electricity/heat supply and pricing to maximize the RCES operator's economic returns.
- Lower-level model controls radiators and electric water heaters to adjust load profiles for cost reduction.
- Solved the bi-level problem using KKT conditions and strong duality, transforming it into a tractable MILP model.
- Case studies show effective supply-demand flexibility coordination, boosting operator profits and reducing consumer costs.



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Focusing on thermal demands including space heating and domestic hot water (DHW), this study develops a hierarchical optimization approach for residential community energy systems (RCES) operating under a multi-energy pricing mechanism. In the upper-level model, the RCES operator utilizes various energy conversion devices to jointly schedule electricity/heat supply and set retail multi-energy prices, aiming to achieve optimal economic returns. Conversely, the lower-level problem explicitly models controllable assets like radiators and electric water heaters, ensuring consumers can satisfy their thermal comfort and water usage needs. Guided by the operator's price signals, residents act as followers to adjust their load profiles, thereby minimizing expenses and realizing integrated demand response (IDR). To solve this bi-level problem, the lower-level constraints are transformed using Karush-Kuhn-Tucker (KKT) conditions and strong duality theory, converting the original model into a tractable single-level mixed-integer linear programming (MILP) formulation. Case studies on a typical RCES indicate that accurately modeling controllable loads facilitates effective IDR. Furthermore, the framework successfully coordinates flexibility on both the supply and demand sides. The results confirm that this strategy yields a win-win outcome, simultaneously boosting the operator's profits and reducing the energy expenditures of residential consumers.

INTRODUCTION

The escalating global challenges of energy security and environmental sustainability demand immediate action.¹ It necessitates a paradigm shift in energy consumption structures and a focused effort on achieving significant energy efficiency improvements.² Achieving this transition requires systematic and effective management with a particular focus on terminal energy consumption.^{3,4} Specifically, the building sector is a dominant energy end-user, accounting for over 30% of global energy usage and approximately one-third of global greenhouse gas emissions.⁵ Due to this significant energy consumption, the building sector holds the greatest potential for achieving energy savings and enhanced efficiency.⁶ In particular, space heating and domestic hot water (DHW) loads constitute the majority of building energy consumption. For instance, within the EU energy landscape, space heating and DHW loads not only account for over 27% of final energy usage and one-fifth of greenhouse gas emissions but also dictate the temporal distribution of heat demand, with space heating and DHW contributing approximately 85% and 15%.⁷ Furthermore, with the accelerating trend of heating electrification, these loads are imposing increasing peak-shaving pressures on the

energy system. However, the large capacity and inherent thermal characteristics of these loads enable them to collectively function as a massive, low-cost virtual battery. This thermal storage capacity is jointly contributed by building structures and dedicated water tanks.⁸ The resulting flexibility strategically positions these loads as high-value controllable resources for terminal energy system.⁹ Therefore, harnessing this inherent flexibility through optimization at the building sector is indispensable for maximizing terminal energy utilization efficiency and supporting a sustainable energy transition.

Demand Response (DR) is acknowledged as a crucial mechanism for effectively harnessing the immense energy consumption flexibility.¹⁰ Traditionally, demand response (DR) has been primarily implemented in the power sector to manage the instability of electrical loads.¹¹ However, driven by strategic frameworks such as the European Green Deal and Energy System Integration, the rapid advancement of heating electrification has significantly expanded the portfolio of controllable resources within the building sector.¹²⁻¹³ This deep coupling of electricity and thermal sectors has catalyzed the evolution from conventional DR to Integrated Demand Response (IDR).¹⁴ In this context, extending DR to thermal systems is a crucial requirement to unlock the full flexibility potential of integrated energy management.

As a local energy supplier and an essential management entity for the building sector, RCES coordinates a wide array of demand-side flexibility resources, making it highly suitable for the implementation of IDR.¹⁵ However, the effective implementation of IDR is constrained by the system's complex physical structure. Specifically, satisfying the buildings' diverse thermal demands (space heating and DHW) requires the deployment of multiple energy conversion units, such as combined heat and power (CHP) units and heat pumps (HPs), to convert and supply energy.¹⁶ This coupling fundamentally leads to the challenge of achieving full-system synergistic optimization and establishing a precise multi-energy pricing mechanism.

In this context, residential consumers and the RCES operator are distinct entities representing conflicting interests. Residential consumers prioritize obtaining high-quality and low-cost multi-energy services from the RCES operator.¹⁷ Therefore, the operator's profit needs to be based on a foundation where the energy economy on the residential consumers is not sacrificed.¹⁸ Accordingly, the energy transactions between residential consumers and the RCES operator must achieve the goal of win-win cooperation and ensure a balance of their respective interests. Specifically, the RCES operator seeks to improve its economic return, while consumers aim to reduce their energy costs and consumption. This hierarchical interaction constitutes a typical Stackelberg game, where the RCES operator acts as the leader setting prices, and consumers act as followers optimizing their usage profiles. Considering

these conflicts and the need for mutual benefit, this study develops a specialized bi-level optimization framework, model, and solution for the RCES operator and the residential consumers.

Motivation and related works

The RCES has been extensively studied as a fundamental unit for decentralized energy management.^{19–21} Existing literature covers a wide range of operational scenarios, optimizing electrical and thermal dispatch under various climatic conditions. In the context of RCES architectures, Talihati, B., et al.²² validated the economic superiority of community energy storage sharing (CESS) over personal storage models by optimizing controllable community loads. To enhance operational efficiency, Mota, B., et al.²³ introduced a genetic algorithm-based load shifting strategy to coordinate appliance flexibility within photovoltaic (PV) energy sharing communities. Extending to multi-vector coupling, Roshani, A.S., et al.²⁴ optimized off-grid RCES integrating hydrogen, heat pumps, and photovoltaic-thermal (PVT) collectors to minimize life cycle costs and environmental impacts. Furthermore, regarding regulatory interactions, Gjorgievski, V.Z., et al.²⁵ revealed the complex synergies between electricity tariffs and storage ownership, emphasizing their impact on the grid interaction of RCES. Nevertheless, a critical research gap remains regarding the distinct attributes of northern regions, where the multifaceted and substantial thermal requirements of buildings dictate the RCES energy consumption patterns. The intricate coupling of inertia-driven space heating and stochastic DHW loads renders precise modeling indispensable for the optimal scheduling of RCES in northern regions.

To address these scheduling challenges within RCES, DR has been widely adopted as a key operational mechanism.²⁶ Initially, research predominantly focused on electrical demand response (EDR). For instance, Kakkar et al.²⁷ highlighted the critical role of optimization algorithms in coordinating the fluctuating charging demands of electric vehicles (EVs) to maintain grid stability. Complementing this, Shen and Chen²⁸ demonstrated that optimized peak-valley pricing (PVP) policies could effectively incentivize residents to shift consumption, reducing peak electrical loads by approximately 9%. However, with the deepening integration of multi-energy carriers, the DR paradigm is progressively evolving from a single electric vector into IDR, which synergistically coordinates multi-energy resources.²⁹ Specifically, Bian et al.³⁰ demonstrated that incorporating the temporal-spatial transferability of data loads into the IDR mechanism could significantly reduce the operating costs of data center clusters by 39.34%. Similarly, Chen et al.³¹ verified that coordinating power and gas demand responses in microgrids effectively enhances system flexibility and balances operational risks under renewable uncertainties. These studies have explored IDR in various types of scenarios. Regarding RCES scenarios, extensive studies have addressed space heating and DHW loads, yet most focus on individual energy management to optimize energy efficiency and consumer costs.^{32–34} Despite these advancements, the flexibility potential inherent in building space heating and DHW loads remains insufficiently exploited in current IDR frameworks to support system level operation. To bridge this gap, this paper proposes a refined multi-energy pricing mechanism to realize IDR in RCES. By responding to the pricing signals through the precise regulation of radiator flow rates and the statuses of electric heaters, the proposed mechanism effectively converts the traditional rigid thermal demands into adjustable resources.

However, the implementation of such a pricing-driven IDR mechanism introduces a complex hierarchical decision-making challenge. The RCES operator acts as a leader, aiming to optimize pricing strategies for system-level benefits, while residential consumer act as followers, responding to these prices to minimize their own costs while maintaining thermal comfort. These objectives are often conflicting and coupled. Conventional centralized optimization methods fail to capture this non-cooperative interaction and the autonomy of demand-side resources. To address similar hierarchical conflicts in energy systems, game-theoretic approaches, particularly the Stackelberg game, and bi-level optimization have been proven effective.³⁵ Liu and Li³⁶ developed a bi-level model to coordinate energy-saving dispatch with regulation-based demand response between generation and load. Focusing on the supply side, Ye et al.³⁷ applied this hierarchical framework to optimize producer bidding strategies by internalizing the non-convex operating characteristics of the market clearing process. Furthermore, Kardakos et al.³⁸ extended this approach to virtual power plants (VPPs), addressing the stochastic optimal offering problem under market uncertainty. It is important

to note that the simultaneous incorporation of space heating and DHW loads in the lower-level problem introduces discrete variables, such as binary ON/OFF statuses, which renders the problem non-convex. This non-convexity makes the conventional method transformations invalid. This paper proposes a bi-level optimization framework and its solution procedure for RCES. The upper level models the RCES operator's pricing decision-making, while the lower level explicitly models the residential consumers' thermal response behaviors (i.e., adjusting radiator flows and heater statuses). This framework and its solution procedure effectively realizes the equilibrium solution that balances the benefits of the RCES operator and residential consumers. Specifically, this win-win equilibrium ensures that neither party can unilaterally improve its objective, where the RCES operator maximizes operational profit through strategic multi-energy pricing, while residential consumers minimize energy expenditures by performing IDR.

Contributions of this paper

This study introduces a hierarchical optimization strategy tailored for RCES operating under multi-energy pricing schemes. Specifically targeting space heating and DHW demands, the model enables the RCES operator to co-optimize energy supply scheduling and retail pricing, while empowering residential consumers to participate in IDR. The following conclusions are observed through numerical case studies:

(1) By leveraging diverse energy conversion technologies, the RCES operator can flexibly switch between energy carriers, thereby unlocking significant flexibility on the supply side. Furthermore, optimal price signals are generated to stimulate IDR from residential consumers, effectively shaping their consumption patterns for better system efficiency.

(2) On the demand side, the thermal inertia of building structures and the storage capability of electric water heaters serve as key flexibility resources. In response to dynamic pricing, residential consumers can minimize utility bills by regulating radiator flow rates and water heater operations, all while ensuring their thermal comfort and DHW requirements.

(3) The proposed bi-level approach effectively facilitates the interaction between the RCES operator and residential consumers. IDR behaviors create a feedback loop that influences the RCES operator's pricing and procurement decisions. Ultimately, the optimization yields an equilibrium solution that optimally reconciles the conflicting economic interests of both the RCES operator and residential customers.

MATERIALS AND METHODS

Bi-level optimization framework for RCES

To characterize the transactional relationship between the RCES operator and residential consumers, this study adopts a bi-level optimization approach, with the day-ahead operational framework depicted in Graphic Abstract (GA).

As shown in GA, the system integrates upstream electricity, gas, and heating networks through various coupling technologies (e.g., CHP units and HPs). Specifically, the proposed framework is designed for residential communities featuring internal energy management and a certain degree of retail pricing autonomy. This operational context allows the RCES operator to function as an intermediary that guides consumer behavior through differentiated internal price signals. The RCES operator has multiple options for supplying energy. It can purchase electricity and heat directly from the upper energy grids. Alternatively, the RCES operator can generate electricity and heating internally. For example, purchased gas can be used to power CHP units, and electricity can be utilized by HPs to produce heat.³⁹ Consequently, the operator can optimally schedule generation and determine retail multi-energy prices for residential consumers, thereby realizing supply-side flexibility.⁴⁰

Generated energy is distributed to residential consumers via local networks. The model assumes the installation of smart controllers on electric water heaters and controllable valves on radiators. These devices enable residential consumers to receive multi-energy price signals and dynamically adjust water flow or DHW equipment status to minimize their costs. Such coordinated regulation of both electrical and heating loads allows residential consumers to actively participate in IDR. This mechanism constitutes the demand-side flexibility within the RCES.

The electrical and thermal energy produced by the RCES operator is distributed to residential consumers properties via the supply networks. This

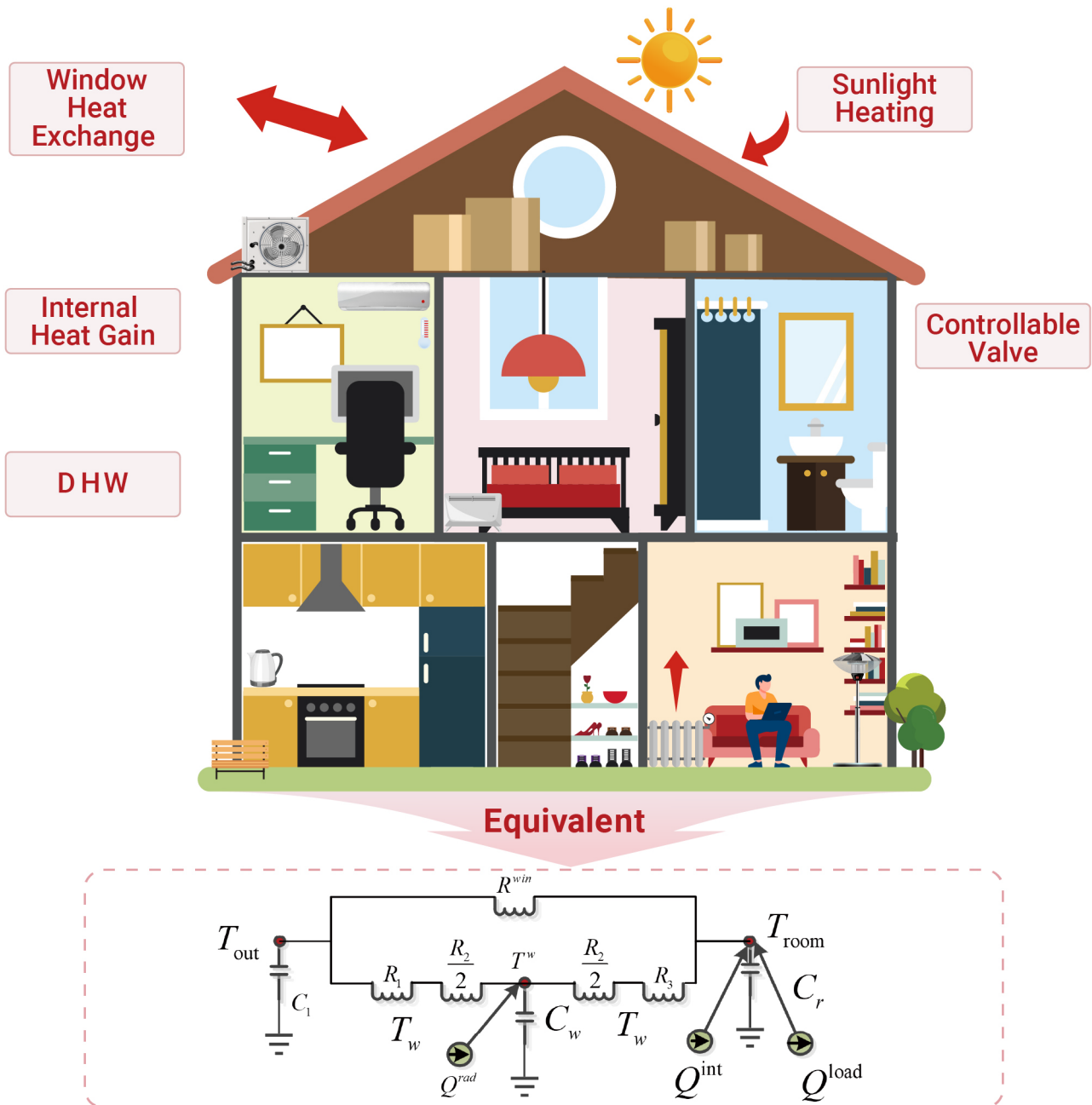


Figure 1. Schematic diagram of the RC model of one zone.

study premises that every radiator is equipped with an electronic control valve, allowing residential consumers to modulate water flow rates in response to heating price signals without compromising their thermal comfort. Specifically, modulating indoor temperature within the comfort range leverages the thermal inertia of the building envelope to shift thermal demand toward low-price periods, thereby providing thermal load flexibility. Similarly, electric water heaters are assumed to feature controllers capable of interpreting electricity price signals to regulate the operational status of DHW loads. Specifically, modulating water temperature within thermal water tanks leverages the heat storage capacity of domestic hot water systems to shift electricity consumption toward low-price periods, thereby providing electrical load flexibility. Upon receipt of multi-energy pricing signals from the RCES operator, these controllers execute IDR tactics designed to lower energy expenses while adhering to the energy usage requirements of residential consumers. Consequently, this mechanism effectively unlocks demand-side flexibility within RCES.

While the RCES operator sets multi-energy prices for residential

consumers, consumers can supply IDR for both electricity and heat in return. This creates a dynamic interaction where the decision-making processes of both the RCES operator and the residential consumers are mutually conditional.⁴¹ As illustrated in GA, the RCES operator is at the upper level, and residential consumers are at the lower level. The RCES operator's objective is to maximize overall profit by optimizing energy acquisition, the usage of conversion equipment, and the retail multi-energy prices offered to lower-level residential consumers. Conversely, the controllers of each residential consumer determine the optimal schedules for space heating and DHW loads' consumption in response to the multi-energy pricing structure set by the upper-level RCES operator. Thus, multi-energy pricing signals influence lower-level space heating and DHW demands, which subsequently impact the energy pricing strategies formulated at the upper level.

Upper level model of the RCES operator

1) Objective function

The objective of the upper-level optimization is to maximize the total operational profit secured by the RCES operator.

$$\max \sum_{t=1}^T \sum_{n=1}^N (C e_t^{\text{sale}} P_{n,t}^{\text{e},L} \Delta t + C h_t^{\text{sale}} P_{n,t}^{\text{h},L} \Delta t) - (C e_t^{\text{buy_Oper}} P_t^{\text{e}} \Delta t + C g_t^{\text{buy_Oper}} P_t^{\text{g}} \Delta t + C h_t^{\text{buy_Oper}} P_t^{\text{h}} \Delta t) \quad (1)$$

2) Operator constraints

In this study, the CHP units are characterized as small-scale, flexible devices distributed throughout the RCES. For the CHP unit under consideration in this case, a fixed conversion efficiency corresponding to the transformation of natural gas into electricity and heat is adopted.

Electric balance:

$$P_t^{\text{e}} + P_t^{\text{chp}} = P_t^{\text{hp}} + \sum_{n=1}^N P_{n,t}^{\text{e},L} \quad (2)$$

$$P_t^{\text{chp}} = \eta_e P_t^{\text{g}} \quad (3)$$

η_e is assigned a value of 0.3.⁴²

Heat balance:

$$H_t^{\text{chp}} + H_t^{\text{hp}} + P_t^{\text{h}} = \sum_{n=1}^N P_{n,t}^{\text{h},L} \quad (4)$$

$$H_t^{\text{chp}} = \eta_h P_t^{\text{g}} \quad (5)$$

$$H_t^{\text{hp}} = \eta_{hp} P_t^{\text{hp}} \quad (6)$$

where N represents the count of residential consumers, η_h is assigned a value of 0.4.³⁹ And η_{hp} is assigned a value of 4.⁴²

Equipment constraints:

To account for the physical capacity limits of the CHP units and heat pumps, the quantities of purchased electricity and gas must be constrained within specific lower and upper bounds, as detailed in Equations (7)–(10).

$$0 \leq P_t^{\text{e}} \leq P_t^{\text{e},\text{max}} \quad (7)$$

$$0 \leq P_t^{\text{g}} \leq P_t^{\text{g},\text{max}} \quad (8)$$

$$0 \leq P_t^{\text{h}} \leq P_t^{\text{h},\text{max}} \quad (9)$$

$$0 \leq P_t^{\text{hp}} \leq P_t^{\text{hp},\text{max}} \quad (10)$$

3) Multi-energy pricing constraints

When setting multi-energy retail prices, the RCES operator must balance its own profit with the economic interests of the residential consumers. Typically, the operator secures energy at a wholesale price lower than the price available to individual residential consumers. Equations (11)–(12) enforce that retail prices remain within optimized boundaries. Furthermore, Equations (13)–(14) ensure fairness by constraining the average multi-energy retail price set by the RCES operator so it does not exceed the average price residential consumers would pay if they purchased directly from the upper-level energy system.

$$C h_t^{\text{sale},\text{min}} \leq C h_t^{\text{sale}} \leq C h_t^{\text{sale},\text{max}} \quad (11)$$

$$C e_t^{\text{sale},\text{min}} \leq C e_t^{\text{sale}} \leq C e_t^{\text{sale},\text{max}} \quad (12)$$

$$\sum_{t=1}^T C h_t^{\text{sale}} / T \leq \sum_{t=1}^T C h_t^{\text{buy_User}} / T \quad (13)$$

$$\sum_{t=1}^T C e_t^{\text{sale}} / T \leq \sum_{t=1}^T C e_t^{\text{buy_User}} / T \quad (14)$$

4) Heating distribution network constraints

This study adopts a regulation strategy based on constant supply temperature with variable mass flow rates.⁴³ Given that the heating distribution network spans a relatively short distance compared to transmission

networks, the thermal dynamics during transmission are treated as insignificant.

Continuity of mass flow:

Mass balance is enforced at every node within the heating network, ensuring that the total inflow equals the total outflow under the assumption of water loss.

$$Y_{\text{HDN}} \dot{m}^p = \dot{m}^n \quad (15)$$

Mass flow rate limit:

To avoid operational issues such as pipeline vibration, the mass flow rates within the water pipes are constrained below their maximum design limits.

$$m_i^{\text{pipe},\text{min}} \leq m_{i,t}^{\text{pipe}} \leq m_i^{\text{pipe},\text{max}} \forall i \in N^{\text{pipe}}, t \in T \quad (16)$$

Pressure loss:

The pressure loss resulting from friction along the pipeline is modeled as being proportional to the square of the mass flow rate, as expressed in Equation (17).

$$p_{n,t} - p_{n+1,t} = \xi_i \cdot (m_{i,t}^{\text{pipe}})^2 \forall i \in N^{\text{pipe}}, n \in N^{\text{node}} \quad (17)$$

$$\xi_i = \frac{8K_i L_i}{d_i^5 \pi^2 \rho} \forall i \in N^{\text{pipe}} \quad (18)$$

Eq. (19) specifies the permissible pressure ranges at the nodes.

$$p_n^{\text{min}} \leq p_{n,t} \leq p_n^{\text{max}} \quad (19)$$

5) Distribution network constraints

This study analyzes a unidirectional, radial distribution network structure. In this setup, bus m acts as the source, while bus $m+1$ serves as the receiving end. The nodal power flow equations are formulated as follows.

$$P_{m+1} = P_m - r_j \frac{P_m^2 + Q_m^2}{V_m^2} - P_{m+1}^{\text{load}} \quad (20)$$

$$Q_{m+1} = Q_m - x_j \frac{P_m^2 + Q_m^2}{V_m^2} - Q_{m+1}^{\text{load}} \quad (21)$$

$$V_{m+1}^2 = V_m^2 - 2(r_j P_m + x_j Q_m) + (r_j^2 + x_j^2) \frac{P_m^2 + Q_m^2}{V_m^2} \quad (22)$$

where $P_m + iQ_m$ represents the apparent power flowing from node m to node $m+1$; V_m represents the nodal voltage magnitude at node m ; $P_{m+1}^{\text{load}} + iQ_{m+1}^{\text{load}}$ represents the apparent power demand at node $m+1$; $r_j + ix_j$ represents the line impedance between node m to $m+1$.

Equation (23) constrains the per-unit voltage deviations along the line within acceptable limits.

$$1 - \varepsilon \leq V_m \leq 1 + \varepsilon \quad (23)$$

ε is generally assigned a value of 0.05.

Lower level model of the residential consumers

1) Objective function

The objective of the lower-level optimization is to minimize the total cost of residential consumers.

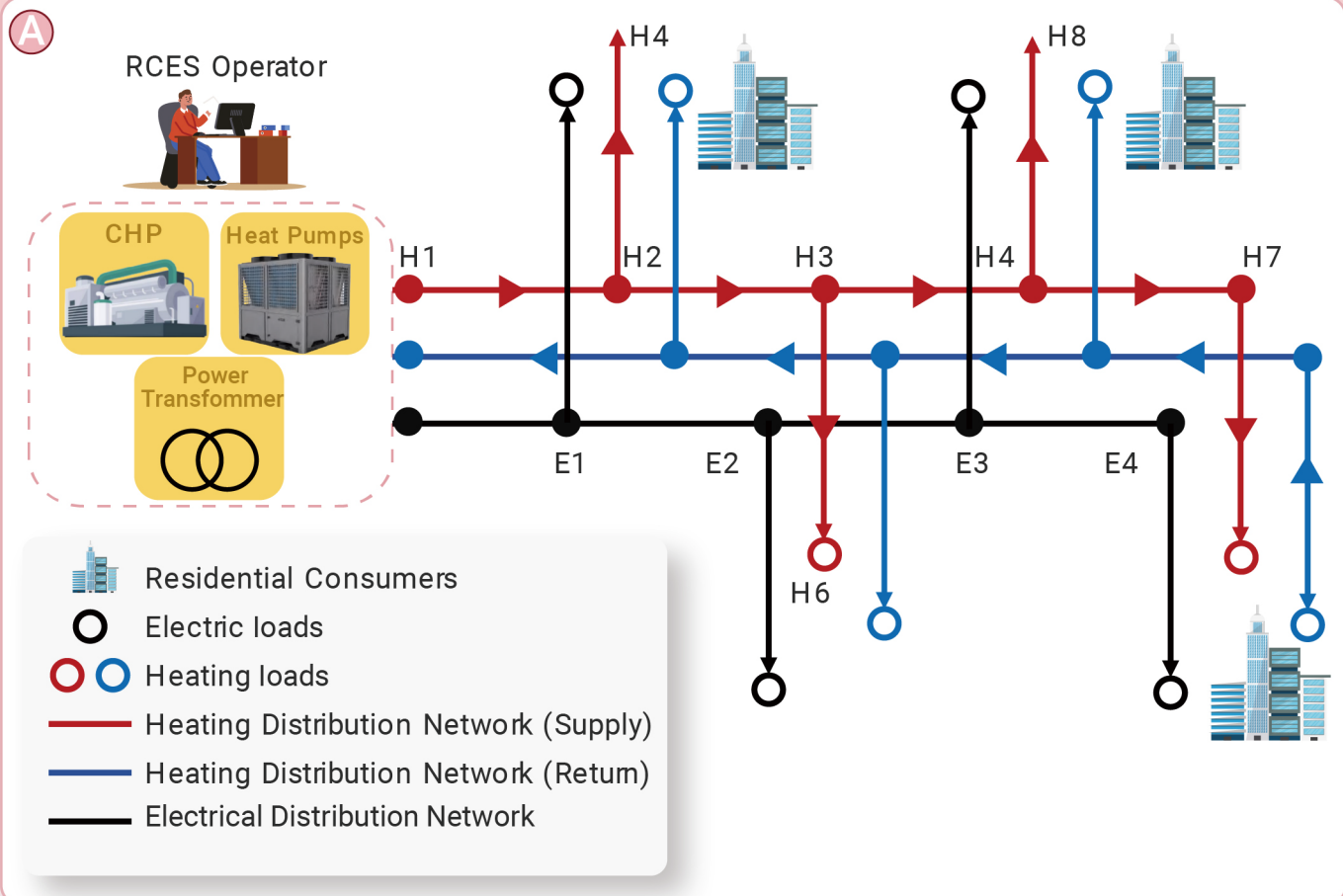
$$\min \sum_{t=1}^T (C e_t^{\text{sale}} \sum_{n=1}^N P_{n,t}^{\text{e},L} \Delta t + C h_t^{\text{sale}} \sum_{n=1}^N P_{n,t}^{\text{h},L} \Delta t) \quad (24)$$

Under the assumption that control strategies are uniform across all zones, the electrical and heating demands of a residential consumer possessing I zones are calculated via Eq. (25) and Eq. (26).

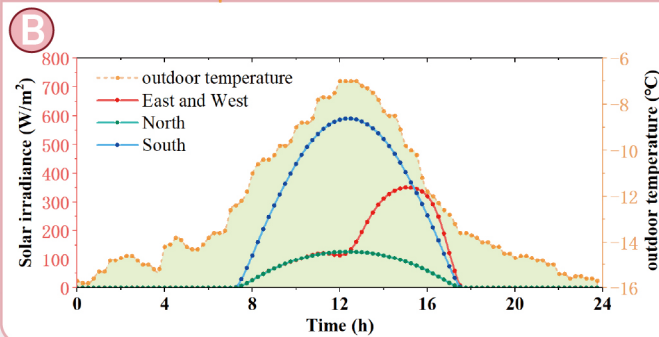
$$P_{n,t}^{\text{e},L} = \sum_{i=1}^I P_i^{\text{e},\text{load}} = \sum_{i=1}^I P_i^{\text{e},\text{DWH}} + P_i^{\text{e},\text{uncontrol}} \quad (25)$$

$$P_{n,t}^{\text{h},L} = \sum_{i=1}^I Q_i^{\text{h},\text{load}} \quad (26)$$

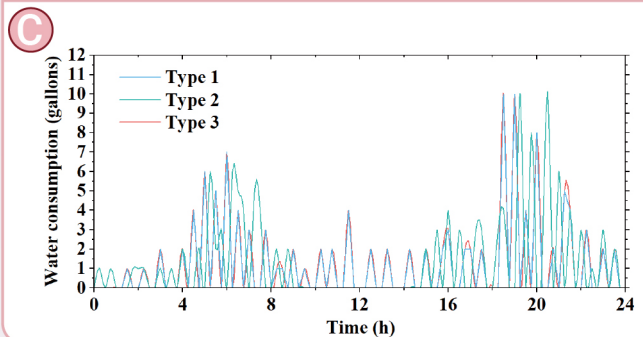
2) Residential consumer's heating load constraints



Weather Data



DHW Demand of Residential Consumers



Energy Purchasing Prices from Upper Energy System

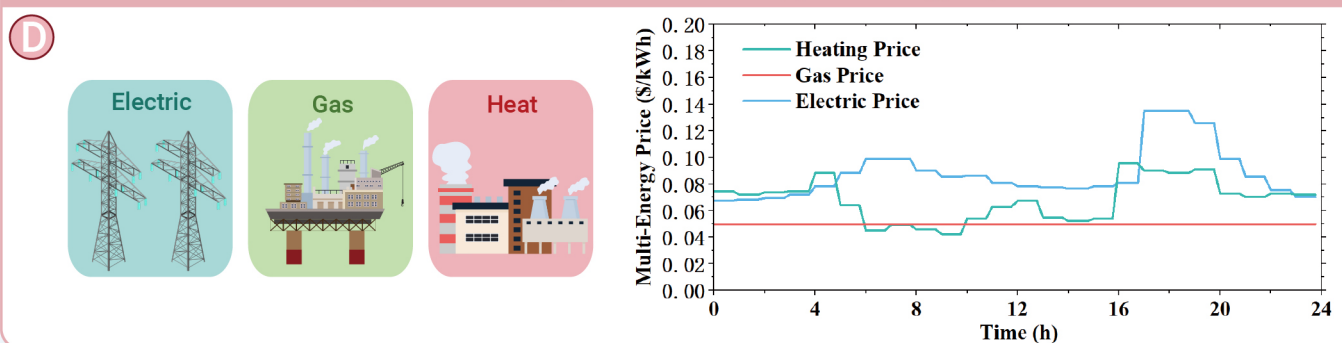


Figure 2. Case setting for network, weather, DHW demand and multi-energy prices.

Thermal balance constraints:

To represent the thermal dynamics of a single zone (both wall and zonal

thermal balance), this study employs an equivalent Resistor-Capacitor (RC) thermal network model, as illustrated in Figure 1.⁴⁴ The formulation of this model is predicated upon three fundamental physical assumptions. These include the lumped parameter assumption for uniform temperature distribution, the linear heat conduction assumption for constant heat transfer coefficients, and the boundary simplification assumption for equivalent thermal resistance and thermal capacitance. This model comprises two distinct node categories: wall nodes and room nodes. These nodes are interconnected via thermal resistances and grounded through thermal capacitors, reflecting the room's capacity to transfer and store heat.

Wall thermal equilibrium constraints are derived within the RC framework by correlating wall temperatures with external heat gains and ambient conditions in Equation (27). Similarly, the thermal balance equation resulting in the zone's indoor air temperature is formulated in Equation (28).

$$\begin{cases} Cw^{1,2}Rw^{1,2}Tw_t^{1,2} = (Cw^{1,2}Rw^{1,2} - 2)Tw_{t-1}^{1,2} \\ + (Tr_{t-1}^1 + Tr_{t-1}^2 + r^{1,2}a^{1,2}Aw^{1,2}Q_{rad}^{1,2})\Delta t\lambda_{1,t} \\ Cw^{1,3}Rw^{1,3}Tw_t^{1,3} = (Cw^{1,3}Rw^{1,3} - 2)Tw_{t-1}^{1,3} \\ + (Tr_{t-1}^1 + Tr_{t-1}^3 + r^{1,3}a^{1,3}Aw^{1,3}Q_{rad}^{1,3})\Delta t\lambda_{2,t} \\ Cw^{1,4}Rw^{1,4}Tw_t^{1,4} = (Cw^{1,4}Rw^{1,4} - 2)Tw_{t-1}^{1,4} \\ + (Tr_{t-1}^1 + Tr_{t-1}^4 + r^{1,4}a^{1,4}Aw^{1,4}Q_{rad}^{1,4})\Delta t\lambda_{3,t} \\ Cw^{1,5}Rw^{1,5}Tw_t^{1,5} = (Cw^{1,5}Rw^{1,5} - 2)Tw_{t-1}^{1,5} \\ + (Tr_{t-1}^1 + Tr_{t-1}^5 + r^{1,5}a^{1,5}Aw^{1,5}Q_{rad}^{1,5})\Delta t\lambda_{4,t} \end{cases} \quad (27)$$

$$Cr'(Tr_t^i - Tr_{t-1}^i) = \sum_{j=1}^4 \frac{Tw_{t-1}^{1,j} - Tr_{t-1}^i}{Rw^{1,j}} + \frac{T_{out} - Tr_{t-1}^i}{Rwin} + c_p * m_{t-1}^i * (T_s - T_r) + Q_{int,t-1}^i + T_{win}A_{win}Q_{rad,t-1}^i\lambda_{5,t} \quad (28)$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ represent dual variables of equality constraints.

The zone's heating requirement (Q_i^{load}) is supplied by a radiator equipped with an electronic control valve, calculated as Eq. (29). This valve provides heating DR by regulating the mass flow rate of the supply water.

$$Q_i^{h,load} = c_p * m_t^i * (T_s - T_r) \quad (29)$$

This study adopts a regulation mode where the supply water temperature remains constant for each residential consumer. To ensure energy conservation and hydraulic stability, the return water temperature in the building's heating network is maintained in a small range via the radiator's valve control. Consequently, for modeling simplicity, the return water temperature is treated as a known constant value.⁴³

Indoor temperature limits:

To preserve the thermal comfort of residential consumers, the indoor temperature is permitted to fluctuate within a specified comfort range:

$$T_{room,min} \leq T_{room} \leq T_{room,max} \cdot \beta_{1,t}^L, \beta_{1,t}^U \quad (30)$$

The comfort range is defined with an upper bound of 24°C and a lower bound of 22°C.

Operating constraints of radiators:

The electronic valve modulates the supply water flow rate, which is constrained by the radiator's maximum design capacity.

$$m_t^{r,min} \leq m_t^i \leq m_t^{r,max} \cdot \beta_{2,t}^L, \beta_{2,t}^U \quad (31)$$

Where $\beta_{1,t}^L, \beta_{1,t}^U, \beta_{2,t}^L, \beta_{2,t}^U$ represent dual variables of inequality constraints.

3) Constraints of residential consumer's DHW load

Thermal balance constraints:

The temperature of DHW load is regulated based on a dead band logic. The boundaries for the water temperature are established by the thermostat setpoint, T_{setr} , and the dead band, δ . Specifically, the upper temperature threshold T_{high} is defined as $T_{setr} + \delta/2$, while the lower threshold T_{low} is $T_{setr} - \delta/2$. The water temperature oscillates between these limits. The heater activates when the tank temperature drops below T_{low} and deactivates once it exceeds T_{high} .

This study utilizes a single-node model to describe the electric-thermal coupling and water temperature dynamics within the tank. Due to the rapid mixing of hot and cold water, the tank temperature is assumed to be uniform, ignoring stratification effects. The change of water temperature in the water tank is shown in Equation (32).

$$T_{t+1}^{DHW} = T_t^{room} - \left(T_t^{room} - \frac{T_t^{DHW}(W - w_t) + T_t^{room}w_t}{W} \right) e^{\frac{-\Delta t}{\tau_{DHW}^{CHW}}} + Q_t^{DHW}R^{DHW} - (Q_t^{DHW}R^{DHW})e^{\frac{-\Delta t}{\tau_{DHW}^{CHW}}} : \lambda_{5,t} \quad (32)$$

$$Q_t^{DHW} = \mu_t^{DHW} Q^{DHW} \quad (33)$$

Similar to the relaxation method for battery storage, where the charging/discharging complementarity is relaxed by considering the operation cost, the binary ON/OFF state of the DHW is relaxed into a continuous power variable in this paper.⁴⁵ Driven by the cost-minimization objective, the linearity of the problem naturally encourages the relaxed variable to converge towards its boundaries (ON/OFF), thereby ensuring the tightness and physical consistency of the relaxed solution. This relaxation transforms the feasible region into a convex set, enabling the derivation of KKT conditions for the bi-level model.

Water temperature limits:

To ensure consumers' comfort level, water temperature of DHW load should be kept within the temperature range defined by the dead zone

$$T_{DHW,min} \leq T_t^{DHW} \leq T_{DHW,max} \cdot \beta_{3,t}^L, \beta_{3,t}^U \quad (34)$$

For this study, the acceptable comfort range is set between a minimum of 35°C and a maximum of 75°C.

Mathematical solution

Reformulate the bi-level model into MPEC model

In the lower-level optimization problem, the multi-energy prices are treated as fixed parameters. Consequently, this sub-problem is strictly continuous and convex. This property allows us to substitute the lower-level objective function and its associated constraints with their Karush-Kuhn-Tucker (KKT) conditions. These KKT conditions are then embedded as constraints within the upper-level optimization problem. Through this transformation, the original bi-level framework is converted into a single-level mathematical problem with equilibrium constraints (MPEC). The lagrangian function is defined in Eq. (35), followed by the KKT conditions for the lower-level model in Eqs. (36)-(43).

1) Lagrange function

$$\begin{aligned} L(Tr_t^i, Tw_t^{1,2}, Tw_t^{1,3}, Tw_t^{1,4}, Tw_t^{1,5}, m_t^i, T_t^{DHW}, Q_t^{DHW}) = \\ \sum_{t=1}^T (Ce_t^{sale} \sum_{n=1}^N P_{n,t}^{e,L} \Delta t + Ch_t^{sale} \sum_{n=1}^N P_{n,t}^{h,L} \Delta t) \\ - \sum_{j \in D_{inequal}} \beta_j h_j + \sum_{k \in D_{equal}} \lambda_k g_k \end{aligned} \quad (35)$$

2) Stationarity conditions

$$\begin{aligned} \frac{\partial L}{\partial T_t^{room}} = \lambda_{1,t} + \lambda_{2,t} + \lambda_{3,t} + \lambda_{4,t} + (Cr * (\sum_{j=2}^5 \frac{1}{Rw^{1,j}}) - \frac{4}{Rwin} - (\sum_{j=2}^5 \frac{1}{Rw^{1,j}}))\lambda_{5,t} \\ - \lambda_{5,t-1} (\frac{1}{Rwin} * Cr * (\sum_{j=2}^5 \frac{1}{Rw^{1,j}})) - \beta_{1,t}^L + \beta_{1,t}^U = 0 \end{aligned} \quad (36)$$

$$\begin{cases} \frac{\partial L}{\partial Tw_t^{1,2}} = (Cw^{1,2} - \frac{2}{Rw^{1,2}})\lambda_{1,t} + \frac{1}{Rwin}\lambda_{5,t} - Cw^{1,2}\lambda_{1,t-1} = 0 \\ \frac{\partial L}{\partial Tw_t^{1,3}} = (Cw^{1,3} - \frac{2}{Rw^{1,3}})\lambda_{2,t} + \frac{1}{Rwin}\lambda_{5,t} - Cw^{1,3}\lambda_{2,t-1} = 0 \\ \frac{\partial L}{\partial Tw_t^{1,4}} = (Cw^{1,4} - \frac{2}{Rw^{1,4}})\lambda_{3,t} + \frac{1}{Rwin}\lambda_{5,t} - Cw^{1,4}\lambda_{3,t-1} = 0 \\ \frac{\partial L}{\partial Tw_t^{1,5}} = (Cw^{1,5} - \frac{2}{Rw^{1,5}})\lambda_{4,t} + \frac{1}{Rwin}\lambda_{5,t} - Cw^{1,5}\lambda_{4,t-1} = 0 \end{cases} \quad (37)$$

$$\frac{\partial L}{\partial m_t^i} = Ch_t^{sale} * (c_p * (T_s - T_r)) + \lambda_{5,t} * (c_p * (T_s - T_r)) - \beta_{2,t}^L + \beta_{2,t}^U = 0 \quad (38)$$

$$\frac{\partial L}{\partial T_t^{DHW}} = \lambda_{1,t} - \lambda_{1,t+1} * (W - w_t) / W * e^{\frac{-\Delta t}{\tau_{DHW}^{CHW}}} - \beta_{3,t}^L + \beta_{3,t}^U = 0 \quad (39)$$

$$\frac{\partial L}{\partial \mu_t^{DHW}} = N * Ce_t^{sale} * Q_t^{DHW} - \lambda_{5,t} Q_t^{DHW} R^{DHW} + \lambda_{6,t} Q_t^{DHW} R^{DHW} * e^{\frac{-\Delta t}{\tau_{DHW}^{CHW}}} = 0 \quad (40)$$

3) Complementary slackness conditions

$$\begin{cases} 0 \leq \beta_{1,t}^L \perp (T_{1,t}^{\text{room}} - T_{1,t}^{\text{room,min}}) \geq 0 \\ 0 \leq \beta_{1,t}^U \perp (T_{1,t}^{\text{room,max}} - T_{1,t}^{\text{room}}) \geq 0 \end{cases} \quad (41)$$

$$\begin{cases} 0 \leq \beta_{2,t}^L \perp (m_t^f - m_t^{f,\min}) \geq 0 \\ 0 \leq \beta_{2,t}^U \perp (m_t^{f,\max} - m_t^f) \geq 0 \end{cases} \quad (42)$$

$$\begin{cases} 0 \leq \beta_{3,t}^L \perp (T_{3,t}^{\text{DHW}} - T_{3,t}^{\text{DHW,min}}) \geq 0 \\ 0 \leq \beta_{3,t}^U \perp (T_{3,t}^{\text{DHW,max}} - T_{3,t}^{\text{DHW}}) \geq 0 \end{cases} \quad (43)$$

Formulation of the MILP model

The MPEC model is non-linear because of the bilinear terms ($Ch_t^{\text{sale}} P_{n,t}^{\text{h,L}}$ and $Ce_t^{\text{sale}} P_{n,t}^{\text{e,L}}$) appearing in the upper-level objective function (1) and the complementary slackness constraints (41)-(43). However, since the lower-level problem is linear, the strong duality theorem is applicable. This theorem posits that the optimal objective value of the primal problem is identical to that of its corresponding dual problem. The duality relationship, as described in Eq. (44), is utilized to resolve the non-linearity.

$$\begin{aligned} \text{Dual_inner}_t &= \sum_{n=1}^T (Ce_t^{\text{sale}} \sum_{n=1}^N P_{n,t}^{\text{e,L}} \Delta t + Ch_t^{\text{sale}} \sum_{n=1}^N P_{n,t}^{\text{h,L}} \Delta t) \\ &= \sum_{t=1}^T \beta_{2,t}^U m_t^{\text{max}} + \sum_{t=1}^T \beta_{1,t}^L T_{1,t}^{\text{room,min}} - \sum_{t=1}^T \beta_{1,t}^U T_{1,t}^{\text{room,max}} + \sum_{t=1}^T (-\frac{T_{1,t}^2}{Rw^{1,2}} \lambda_{1,t}) \\ &\quad + \sum_{t=1}^T (-r^{1,2} a^{1,2} Aw^{1,2} Qrad^{1,2} \lambda_{1,t}) + \sum_{t=1}^T (-\frac{T_{1,t}^3}{Rw^{1,3}} \lambda_{2,t}) \\ &\quad + \sum_{t=1}^T (-r^{1,3} a^{1,3} Aw^{1,3} Qrad^{1,3} \lambda_{2,t}) + \sum_{t=1}^T (-\frac{T_{1,t}^4}{Rw^{1,4}} \lambda_{3,t}) \\ &\quad + \sum_{t=1}^T (-r^{1,4} a^{1,4} Aw^{1,4} Qrad^{1,4} \lambda_{3,t}) + \sum_{t=1}^T (-\frac{T_{1,t}^5}{Rw^{1,5}} \lambda_{4,t}) \\ &\quad + \sum_{t=1}^T (-r^{1,5} a^{1,5} Aw^{1,5} Qrad^{1,5} \lambda_{4,t}) \\ &\quad + \sum_{t=1}^T (-\frac{T_{1,t}^{\text{out}}}{Rw^{\text{in}}} \lambda_{5,t}) + \sum_{t=1}^T (-Q_{1,t}^{\text{int}} \lambda_{5,t}) + \sum_{t=1}^T (-T^{\text{win}} A^{\text{win}} Q_{1,t}^{\text{win}} \lambda_{5,t}) \\ &\quad + \sum_{t=1}^T \beta_{1,t}^L T_{1,t}^{\text{DHW,min}} - \sum_{t=1}^T \beta_{1,t}^U T_{1,t}^{\text{DHW,max}} + \sum_{t=1}^T (T_{1,t}^{\text{room}} \lambda_{1,t}) \\ &\quad + \sum_{t=1}^T (T_{1,t}^{\text{room}} e^{\frac{-\Delta t}{R^{\text{DHW}} C^{\text{DHW}}}} \lambda_{1,t}) + \sum_{t=1}^T (\frac{T_{1,t}^{\text{room}} W_t}{W} e^{\frac{-\Delta t}{R^{\text{DHW}} C^{\text{DHW}}}} \lambda_{1,t}) \end{aligned} \quad (44)$$

By leveraging the strong duality equality, the non-linear components within the upper-level objective function are substituted with linear expressions. The revised linear objective function for the upper-level model is presented in Eq. (45).

$$\max \sum_{t=1}^T [\text{Dual_inner}_t - (Ce_t^{\text{buy_Oper}} P_t^e \Delta t + Cg_t^{\text{buy_Oper}} P_t^g \Delta t + Ch_t^{\text{buy_Oper}} P_t^h \Delta t)] \quad (45)$$

Furthermore, the Big-M method is employed to linearize the complementary slackness conditions. This involves introducing auxiliary binary variables to transform the constraints into a linear format, as shown below:

$$0 \leq \beta_{1,t}^L \leq M_1 \theta_{1,t}^L \quad (46)$$

$$0 \leq T_{1,t}^{\text{room}} - T_{1,t}^{\text{room,min}} \leq M_1 (1 - \theta_{1,t}^L) \quad (47)$$

$$0 \leq \beta_{1,t}^U \leq M_1 \theta_{1,t}^U \quad (48)$$

$$0 \leq T_{1,t}^{\text{room,max}} - T_{1,t}^{\text{room}} \leq M_1 (1 - \theta_{1,t}^U) \quad (49)$$

$$0 \leq \beta_{2,t}^L \leq M_2 \theta_{2,t}^L \quad (50)$$

$$0 \leq m_t - m_t^{\min} \leq M_2 (1 - \theta_{2,t}^L) \quad (51)$$

$$0 \leq \beta_{2,t}^U \leq M_2 \theta_{2,t}^U \quad (52)$$

$$0 \leq m_t^{\max} - m_t \leq M_2 (1 - \theta_{2,t}^U) \quad (53)$$

$$0 \leq \beta_{3,t}^L \leq M_3 \theta_{3,t}^L \quad (54)$$

$$0 \leq T_{3,t}^{\text{DHW}} - T_{3,t}^{\text{DHW,min}} \leq M_3 (1 - \theta_{3,t}^L) \quad (55)$$

$$0 \leq \beta_{3,t}^U \leq M_3 \theta_{3,t}^U \quad (56)$$

$$0 \leq T_{3,t}^{\text{DHW,max}} - T_{3,t}^{\text{DHW}} \leq M_3 (1 - \theta_{3,t}^U) \quad (57)$$

where $\theta_{1,t}^L, \theta_{1,t}^U, \theta_{2,t}^L, \theta_{2,t}^U, \theta_{3,t}^L, \theta_{3,t}^U$ represents the auxiliary binary variables. M_1, M_2, M_3 represents sufficiently large constants.

Regarding the network constraints, the hydraulic and power flow equations (Equation (17) and Equations (20)-(22)) are non-linear and non-convex. To address this, piecewise linear functions are utilized to approximate the non-linear behavior. Specifically, the non-linear flow rate term in Equation (17) is discretized into Q segments along the abscissa. The number of segments must be calibrated based on the actual system characteristics to ensure high solution quality.

The corresponding slopes and ordinates for these segments are calculated via Equations (58)-(59).

$$y_i^q = 0 \quad x_i^q = 0 \quad q = 1 \quad (58)$$

$$y_i^q = (x_i^q)^2 \quad x_i^q = \frac{m_i^{\text{pipe,max}} \cdot q}{Q} \cdot k_i^q = \frac{y_i^q - y_i^{q-1}}{x_i^q - x_i^{q-1}} \cdot q \geq 2 \quad (59)$$

Through this segmentation, the original non-linear curves are approximated by a system of linear equations, as detailed in Equation (60). The constraints governing these piecewise linear segments are defined in Equations (61)-(62).

$$p_{n,t} - p_{n+1,t} = \xi_i \cdot \sum_{q=1}^Q [(A_{i,t}^{\text{pipe}} - x_i^q \delta_{i,t}^q) k_i^q + y_i^q \delta_{i,t}^q] \quad (60)$$

$$\delta_{i,t}^q x_i^q \leq A_{i,t}^{\text{pipe}} \leq \delta_{i,t}^q x_i^{q+1} \quad (61)$$

$$\sum_{q=1}^Q \delta_{i,t}^q = 1 \quad (62)$$

where $A_{i,t}^{\text{pipe}}$ represents the numerical value of the mass flow, $\delta_{i,t}^q$ is auxiliary binary variables, which represents whether the value of $A_{i,t}^{\text{pipe}}$ is in the abscissa of q .

The electrical power flow equations (Equations (20)-(22)) are linearized by applying standard simplifying assumptions, a method widely accepted and validated in electricity distribution system analysis.⁴⁶ Following this linearization, the distribution network constraints are updated in Equations (63)-(65).

$$P_{n+1} = P_n - P_{n+1}^{\text{load}} \quad (63)$$

$$Q_{n+1} = Q_n - Q_{n+1}^{\text{load}} \quad (64)$$

$$V_{n+1} = V_n - \frac{r_f P_n + x_f Q_n}{V_0} \quad (65)$$

With the objective function and all constraints now fully linearized, the entire problem is transformed into a standard MILP model. This model is implemented using the YALMIP toolbox within the MATLAB environment and solved using the CPLEX solver.

RESULTS

Case study

Case setting

The configuration depicted in Figure 2A serves as the experimental system for this study. The architectural model is based on a single residential structure. Every residential consumer unit comprises four distinct zones, with dimensions of 6m in length, 6m in width, and 3m in height per zone. The

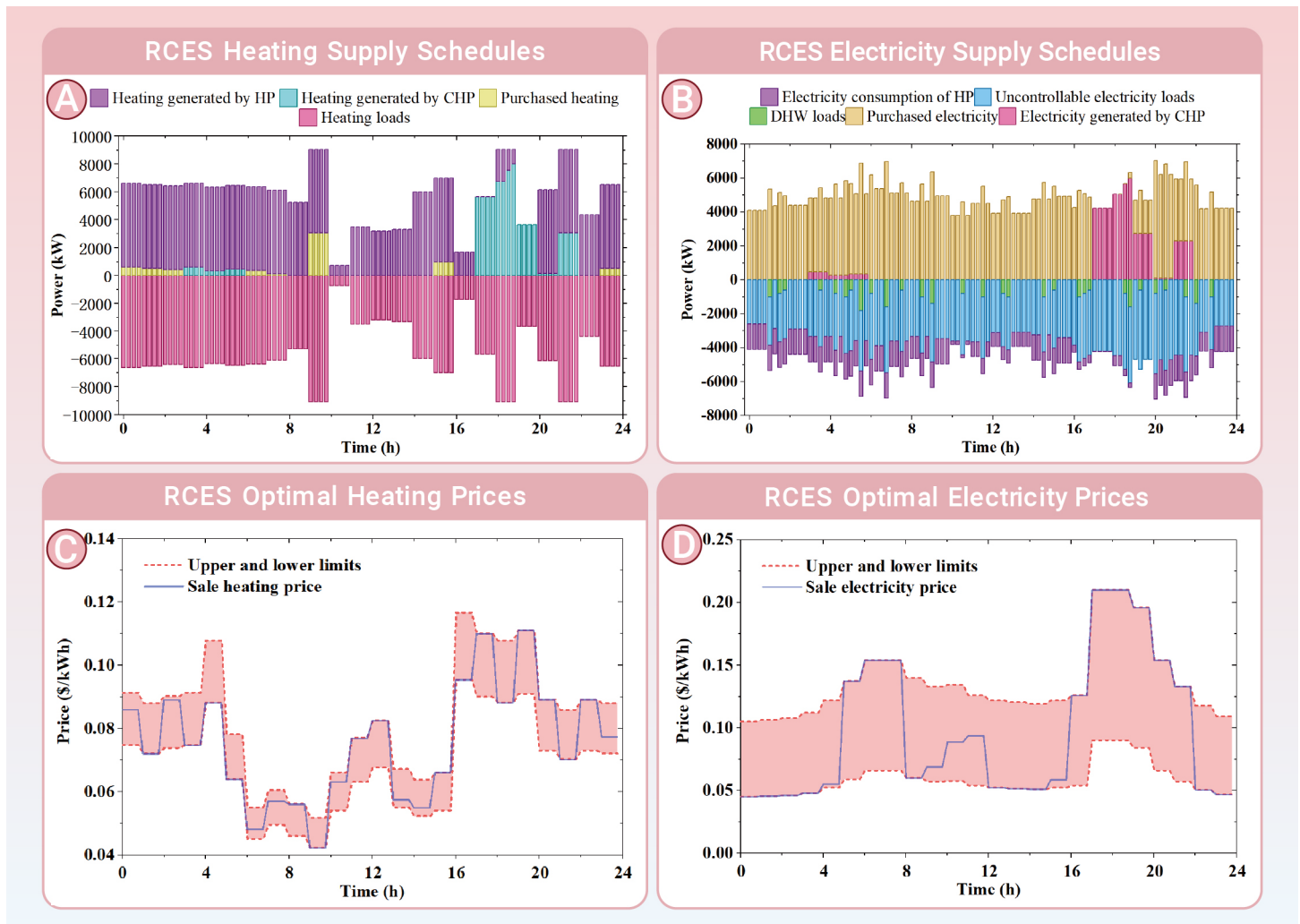


Figure 3. Case setting for network, weather, DHW demand and multi-energy prices.

building consists of 20 stories, accommodating two residential consumers on each floor.⁴⁷ The community aggregation is composed of 30 such identical buildings. Regarding network topology, groups of 10, 8, 6, and 6 buildings are linked to electricity distribution nodes 1, 2, 3, and 4, and correspondingly to heating distribution nodes 4, 6, 8, and 9. Figure 2B illustrates the profiles for solar irradiance and outdoor ambient temperature used in the simulation.⁴⁸⁻⁴⁹

Different types of residential consumers have different hot water consumption behaviors. And the hot water consumption behavior is one of the main factors that affect the demand response of water heaters. RCES operators should consider the electricity loads of all building consumers when setting electricity prices. In this context, this paper simulates the hot water demand curves of three types of consumers, as shown in Figure 2C: Type 1 corresponds to elderly users. Type 2 corresponds to office workers. Type 3 combines Type 1 and Type 2, which corresponds to mixed consumers between these two situations.⁵⁰ In this paper, assuming that 1/3, 1/4 and 5/12 of consumers in each building correspond to the water consumption of Type 1, Type 2 and Type 3. Additionally, Figure 2D depicts the operator's energy procurement costs, which are set at a rate 10% below the price consumers would pay directly to the upper energy systems.

It should be noticed that the model's parameters are formulated as reconfigurable inputs. This parameterized structure allows the case setup to be flexibly scaled and adapted to represent diverse energy systems with varying structure, load compositions, and climatic conditions.

Simulation results

1) Results of the RCES operator

Figure 3 displays the optimized schedules for energy supply and the corresponding multi-energy pricing set by the RCES operator.

Referring to Figure 3A, the operator prioritizes HPs for thermal supply during the majority of the day, owing to their superior efficiency compared to CHP units. During peak heating load intervals where HP capacity is insufficient, the operator must secure alternative heating sources. If external heating prices drop significantly below gas prices, the strategy shifts towards purchasing heat directly from the upper energy system. Conversely, when heating is more expensive than gas and load disparities are minimal, the preference of RCES operator changes. Under these conditions, the RCES operator opts to buy gas to co-generate both heat and power.

Similarly, due to the CHP's relatively lower electrical efficiency, the operator primarily satisfies consumer demand by purchasing electricity from the grid. However, during periods of high electricity prices, as indicated in Figure 3B, the RCES operator utilizes gas to drive the CHP for power and heat generation.

Figure 3C & D present the optimized retail multi-energy prices for electricity and heating, which fluctuate within their designated permissible ranges.

Figure 3 A & C reveal that despite residential consumer heating consumption, the RCES operator's procurement costs from the upper system remain moderate. Consequently, setting the sales price at the lower bound does not negatively impact the RCES operator's profitability. In contrast, when the operator must purchase expensive electricity from the grid to satisfy heating loads, costs increase. To safeguard profit margins under high production costs, the heating sales price is adjusted to its upper limit.

Similarly, as depicted in Figure 3 B & D, the operator raises electricity prices to the maximum cap during periods of high consumer demand. Although

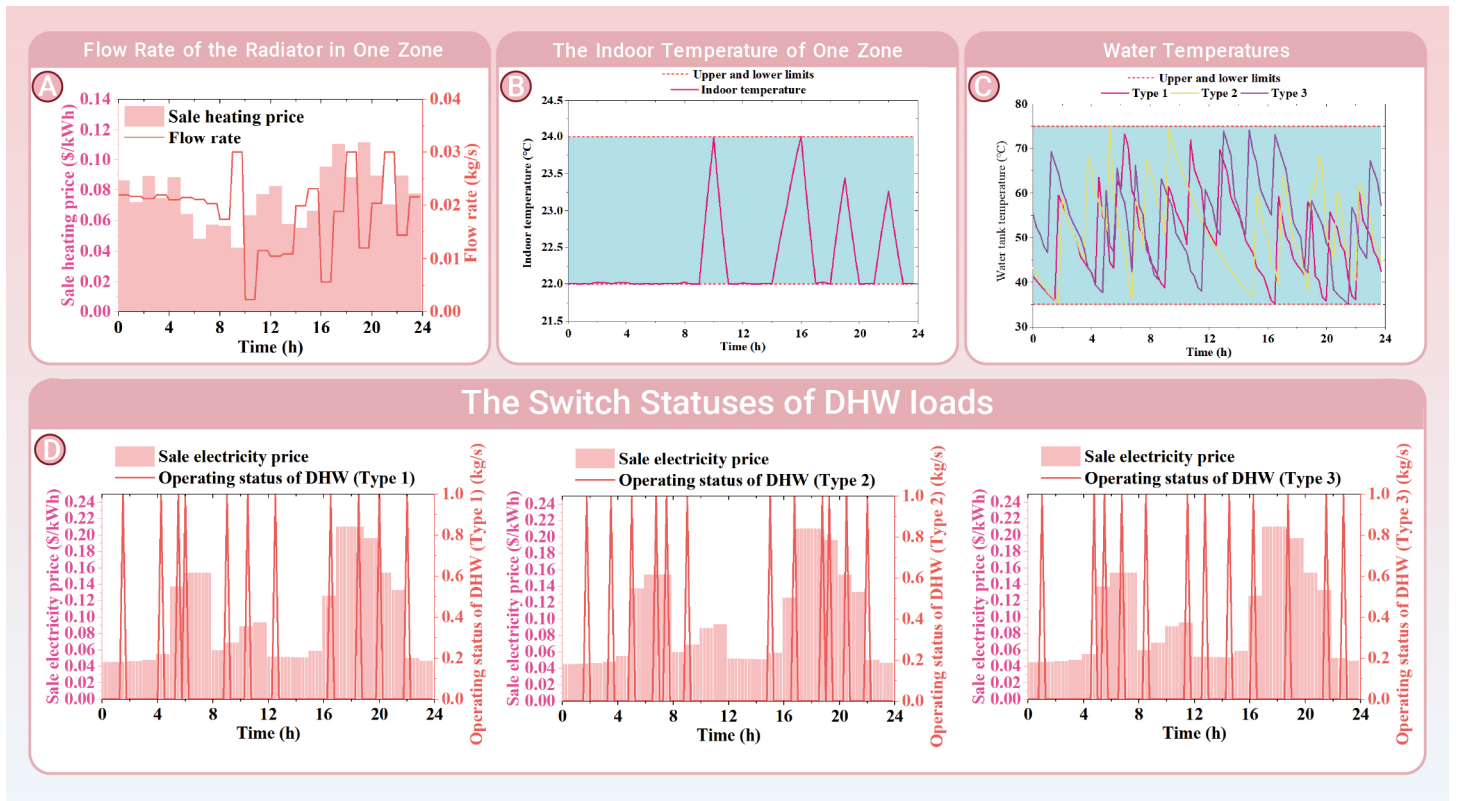


Figure 4. Results of residential consumers.

DHW heaters offer load-shifting capability, these intervals remain peak consumption times with high external purchase costs. Therefore, the operator maximizes the price to the upper bound to ensure financial returns. Nevertheless, to comply with daily average price constraints, the RCES operator is compelled to select the lower price limit during off-peak intervals.

2) Results of residential consumers

Driven by outdoor temperature and the selling price of heating, the associated flow rate of one zone is presented in Figure 4A, while the corresponding indoor and outdoor temperatures are depicted in Figure 4B.

Leveraging the building's thermal inertia, residential consumers contribute to heating DR by modulating their radiator flow rates. As demonstrated in Figure 4A, residential consumers increase flow rates to pre-store thermal energy during periods of lower heating prices. This stored heat is utilized during price peaks, thereby reducing overall heating expenditures. Conversely, in scenarios with minimal price volatility, the DR behavior of residential consumers becomes less pronounced. Furthermore, flow rate adjustments impact indoor temperatures, while the permissible temperature bounds simultaneously constrain flow rate modifications.

Under the influence of residential consumers' hot water demand and sale electricity price, the switch statuses of DHW loads and water temperatures of three types of residential consumers, as shown in Figure 4C & D. Residential consumers can provide EDR to RCES operator by controlling the statuses (ON/OFF) of DHW loads. As shown in Figure 4C, the sale electricity price is high, and this period is also the peak time for residential consumers' water consumption in the morning. In this case, all three types of consumers choose to heat the water heater in advance to avoid heating the water heater during peak hours. Another sale electricity price peak is from 18:00 to 20:00. As shown in Figure 4C, residential consumer type 1 and type 3 consume more hot water from 19:00 to 20:00. Since consumers use much more water for bathing at night than in the morning, the water heater still has to be opened to meet the residential consumers' larger water demand even if the sale electricity price is high. In summary, taking advantage of the demand response capability of water heaters can reduce consumers' electricity loads and cost to a certain extent. However, the demand response capacity of electric water heater is also related to the consumer's DHW demand. The DR of water heaters to the retail electricity price is not evident during periods when consumers have relatively high DHW demand.

3) Comparative studies

To validate the proposed bi-level optimization framework, three distinct scenarios were established.

Scenario I: The bi-level optimization method proposed in this paper, considering both dynamic multi-energy pricing and IDR.

Scenario II: The RCES operator sets fixed sale energy prices within a specified range, implying that residential consumers passively accept prices without the bi-level interaction. This scenario is used to compare with Scenario I to validate the equilibrium solution of the multi-energy pricing strategy.

Scenario III: Residential consumers strictly follow their baseline demand for space heating and DHW loads without providing IDR. This scenario is compared with Scenario I to demonstrate the necessity and benefits of the electricity and heating DR.

Figure 5 presents a comparative analysis of Scenarios I and II, illustrating the RCES operator's profit versus residential consumers' energy costs across 24 price groups.

As indicated in Figure 5, a direct correlation exists where higher residential consumers' energy costs yield increased RCES operator profits. Specifically, higher prices favor the RCES operator, whereas lower prices benefit the residential consumers. For instance, under the highest fixed price setting in Scenario II, RCES operator profits rise by 26% and residential consumers' costs by 11% compared to Scenario I. Conversely, at the lowest price point, residential consumers' costs drop by 11%, but RCES operator profits simultaneously decline by 26% relative to Scenario I. Thus, the optimized multi-energy pricing (marked by the red dot) represents a central equilibrium solution that balances the conflicting economic interests of both parties.

Table 1 details the settings for the comparison between Scenarios I and III, while Table 2 outlines the resulting financial outcomes for both stakeholders.

Analysis of Table 2 yields the following insights:

(a) A comparison of Scenario I with III-1 and III-2 illustrates the advantages of IDR participation for residential consumers. While RCES operator profits and heating costs are similar in Scenarios I and III-1, the latter results in a 5.6% increase in residential consumers' electricity costs. Similarly, in Scenario III-2, although electricity costs align with Scenario I, heating costs surge by approximately 5.4%. Furthermore, the total energy costs of residential consumers in scenario III-1 and III-2 are also higher than in those of Scenario I. Since residential consumers cannot participate in IDR, residential

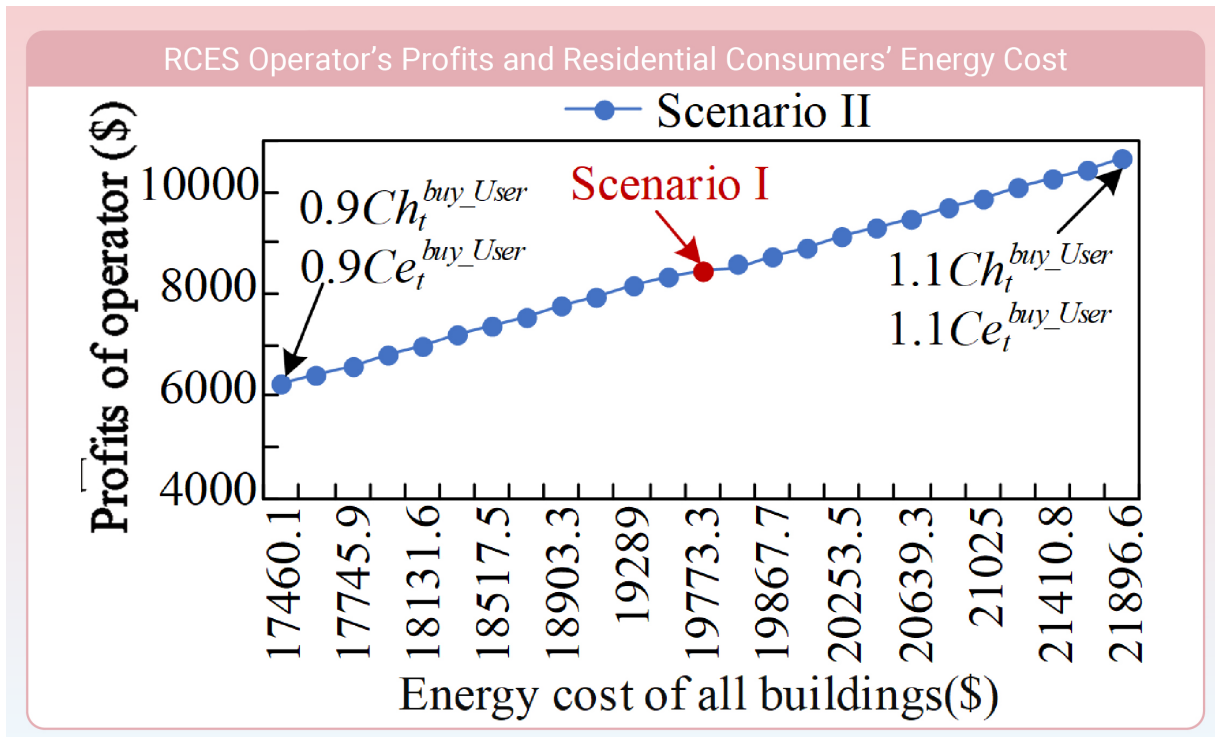


Figure 5. RCES operator's profits and residential consumers' energy cost in scenario I and scenario II.

consumers' energy costs will increase when the RCES operator's profits are determined. Therefore, it can be seen from the results that considering the residential consumers' electricity and heating DR capabilities can reduce the energy costs of residential consumers to a certain extent.

(b) Comparing Scenario I with III-3 demonstrates the benefits of IDR for the RCES operator. Although residential consumers' electricity and heating costs are almost same in scenario I and scenario III-3, RCES operator experiences lower profits (around 6.2% lower for RCES operator) in scenario III-3. In this case, residential consumers cannot reduce energy consumption through IDR. When residential consumers' energy costs need to be reduced the same as scenario I, RCES operator should reduce the multi-energy price, and the RCES operator's profits will decrease. Therefore, considering the residential consumers' electricity and heating DR capabilities can also give RCES operator more flexibility in multi-energy pricing strategies to a certain extent, thereby increasing their profits.

CONCLUSION

This study establishes a bi-level optimization framework for RCES to coordinate multi-energy pricing with consumer's IDR behaviors. Specifically, the model incorporates multi-energy pricing of RCES operator to incentivize electricity and heating load adjustments of residential consumers. The following conclusions are observed through numerical case studies:

(1) **Supply-side flexibility and pricing:** The RCES operator effectively unlocks supply-side flexibility by dynamically selecting optimal energy conversion pathways (e.g., via CHP or HPs). Furthermore, the optimized multi-energy pricing strategy serves as a crucial signal to draw out IDR from residential consumers, thereby reshaping consumption profiles to achieve system-wide optimality.

(2) **Demand-side flexibility and IDR:** The space heating and DHW loads can provide demand side flexibility in RCES, exhibit significant flexibility potential. By modulating the flow rate of residential consumers' radiators and the status of DHW loads (ON/OFF) in response to retail price signals, residential consumers can reduce energy expenditures while maintaining their thermal comfort and energy requirements.

(3) **Equilibrium and coordination:** The proposed bi-level approach effectively captures the non-cooperative interaction between the RCES operator and residential consumers. The results demonstrate that the interactive pricing mechanism leads to an equilibrium, where the RCES operator's pricing decisions and residential consumers' usage shifts are perfectly aligned. This

equilibrium facilitates a "win-win" outcome, enhancing RCES operator profits while reducing residential consumers' costs compared to passive operation modes.

DISCUSSION

To further enhance the strategic value and practical applicability of the proposed RCES flexibility assessment, future efforts will prioritize the integration of operational uncertainties and the multi-time scale characterization of consumer behavior, ensuring the methodology aligns with the increasing complexity of modern RCES.

Regarding operational reliability, research will focus on internalizing the inherent volatility of the RCES environment. Factors such as stochastic fluctuations in ambient temperature and the randomness of user energy consumption can lead to non-negligible deviations in flexibility potential. By incorporating these uncertain parameters, the updated framework will provide a more resilient assessment of electrical and thermal load flexibility under fluctuating operating conditions.

Beyond day-ahead operational framework, the evolution of IDR across differentiated time scales presents a significant research frontier. In the medium term, the focus will be on behavioral elasticity, where cost trends guide consumers toward lifestyle-based adjustments. In the long term, research will address structural flexibility, analyzing how price signals incentivize capital investments in building insulation or storage capacity. Investigating these multi-scale response mechanisms will enable RCES operators to accurately predict baseline shifts and optimize infrastructure planning.

Nomenclature

Abbreviations

CESS	Community Energy Storage Sharing
CHP	Combined Heat and Power
DHW	Domestic Hot-Water
DR	Demand Response
EDR	Electrical Demand Response
EV	Electric Vehicle
HP	Heat Pump
IDR	Integrated Demand Response
KKT	Karush-Kuhn-Tucker
MILP	Mixed-Integer Linear Programming
PV	Photovoltaic

Table 1. Case setting in scenario I and scenario III.

Scenario	RCES operator's sale multi-energy price		Consumers participate in IDR
	Heating price	Electricity price	
I	Optimized	Optimized	✓
III-1	$0.835Ch_t^{buy_Oper}$	$1.111Ce_t^{buy_Oper}$	X
III-2	$0.879Ch_t^{buy_Oper}$	$1.051Ce_t^{buy_Oper}$	X
III-3	$0.835Ch_t^{buy_Oper}$	$1.051Ce_t^{buy_Oper}$	X

PV-T Photovoltaic-Thermal
PVP Peak-Valley Pricing
RCES Residential Community Energy System
VPP Virtual Power Plant

Indices and Sets

T Index of time
 N Number of residential consumers
 N^{pipe} Number of water pipeline
 N^{node} Number of nodes
 M Number of nodes in the distribution network
 J Number of branches in the distribution network
 I Number of zones of residential consumers

Parameters

$Ce_t^{buy_Oper}$, Operator's purchasing prices of electricity/gas/heating from upper energy systems at time t
 $Cg_t^{buy_Oper}$
 $Ch_t^{buy_Oper}$
 η_e, η_h Conversion efficiency of gas into electricity/heating through combined heat and power (CHP)
 η_{hp} Conversion efficiency of electricity into heating through heat pump (HP)
 $P_t^{e,max}$ The maximum electrical power sold by the upper energy system to RCES operator at time t
 $P_t^{g,max}$ The maximum gas sold by the upper energy system to RCES operator at time t
 $P_t^{h,max}$ The maximum heating power sold by the upper energy system to RCES operator at time t
 $P_t^{hp,max}$ The maximum electricity consumption of HP
 $Ch_t^{sale,min}$ The minimum heating price sold by the RCES operator at time t
 $Ch_t^{sale,max}$ The maximum heating price sold by the RCES operator at time t
 $Ce_t^{sale,min}$ The minimum electrical price sold by the RCES operator at time t
 $Ce_t^{sale,max}$ The maximum electrical price sold by the RCES operator at time t
 Y_{HDN} Incidence matrix of heating distribution network
 $m_l^{pipe,max}$ The maximum mass flow rate in pipe l
 $m_l^{pipe,min}$ The minimum mass flow rate in pipe l
 ξ_l The hydraulic resistance coefficient of pipeline l
 κ_l The friction coefficient of pipeline l
 L_l The length of pipeline l
 d_l The inner diameter of pipeline l
 ρ The density of the heat medium (water)
 p_n^{max} The maximum pressure at node n
 p_n^{min} The minimum pressure at node n
 r_j The resistance of the branch j
 x_j The reactance of the branch j
 Q_i^{load} The reactive power load at node n
 ε The maximum allowable voltage deviation
 $P_i^{uncontrol}$ The uncontrollable load of zone i
 Cw The heat capacity of the wall
 Rw The thermal resistance of the wall
 r The identifier of the sunlit wall
 α The radiative heat absorption coefficient of the wall between the room and outdoor
 Aw The area of the wall
 Q_{rad} The solar irradiance
 $Rwin$ The thermal resistance of the window

Table 2. Benefits of the RCES operator and energy cost of residential consumers.

Scenario	Profits of the operator (\$)	Electricity costs of residential consumers (\$)	Heating costs of residential consumers (\$)	Energy costs of residential consumers (\$)
I	8468.1	9718.8	10054.5	19773.3
III-1	8470.6	10260.4	10049.3	20309.7
III-2	8470.8	9715.0	10590.7	20305.7
III-3	7941.3	9715.0	10049.3	19764.3

c_p The specific heat capacity of the water
 T_s, T_r Supply and return water temperature
 $Q_{int,t}$ Internal heat gain of the zone at time t
 T_{win} The transmittance of window
 A_{win} The window area
 $T_{room,max}$ The maximum temperature of residential consumer room
 $T_{room,min}$ The minimum temperature of residential consumer room
 $m^{r,max}$ The maximum mass flow rate of the radiator at time t
 $m^{r,min}$ The minimum mass flow rate of the radiator at time t
 W The water tank volume of DHW
 w_t The residential consumer's water consumption at time t
 R^{DHW} The thermal resistance of DHW
 C^{DHW} The heat capacity of DHW
 Q^{DHW} The operating heat power of DHW
 $T^{DHW,max}$ The maximum water temperature of DHW
 $T^{DHW,min}$ The minimum water temperature of DHW
 h_j The inequality constraint j
 g_k The equality constraint k

Variables

Ce_t^{sale} RCES operator's sale electric price at time t
 Ch_t^{sale} RCES operator's sale heating price at time t
 $P_{n,t}^{e,L}$ Electric load of residential consumer n at time t
 $P_{n,t}^{h,L}$ Heating load of residential consumer n at time t
 P_t^e The electrical power sold by the upper grid to RCES operator at time t
 P_t^g The gas sold by the upper grid to RCES operator at time t
 P_t^h The heating power sold by the upper grid to RCES operator at time t
 P_t^{chp} The electrical power generation of CHP at time t
 P_t^{hp} The electrical power consumption of HP at time t
 H_t^{chp} The heating power generation of CHP at time t
 H_t^{hp} The heating power generation of HP at time t
 $\dot{m}^p$ The vector of mass flow rates in pipes
 $\dot{m}^n$ The vector of nodal mass flow injections
 $m_{l,t}^{pipe}$ The mass flow rate in pipe l at time t
 $p_{n,t}$ The pressure at node n at time t
 P_i The active power flow from node i to node $i+1$
 Q_i The reactive power flow from node i to node $i+1$
 V_i The voltage magnitude at node i
 P_i^{load} The active power load at node n
 $P_i^{e,load}$ The electrical load of zone i
 P_i^{DHW} The DHW load of zone i
 $Q_i^{h,load}$ The heating load of zone i
 Tw_t The temperature of the wall at time t
 Tr_t The temperature of the room at time t
 m_t^r The mass flow rate of the radiator at time t
 λ The dual variables of equality constraints
 β The dual variables of inequality constraints
 T_t^{DHW} The water temperature of DHW at time t
 μ_t^{DHW} The switch status of DHW at time t
 Q_t^{DHW} The operating heat power of DHW at time t

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DECLARATION OF INTERESTS

The authors declare no competing interests.

DATA AND CODE AVAILABILITY

Data are available from the corresponding author upon reasonable request.