



A Unified Value Signal Driven Strategy for Coordinated Multi-Energy Optimization in Building Clusters

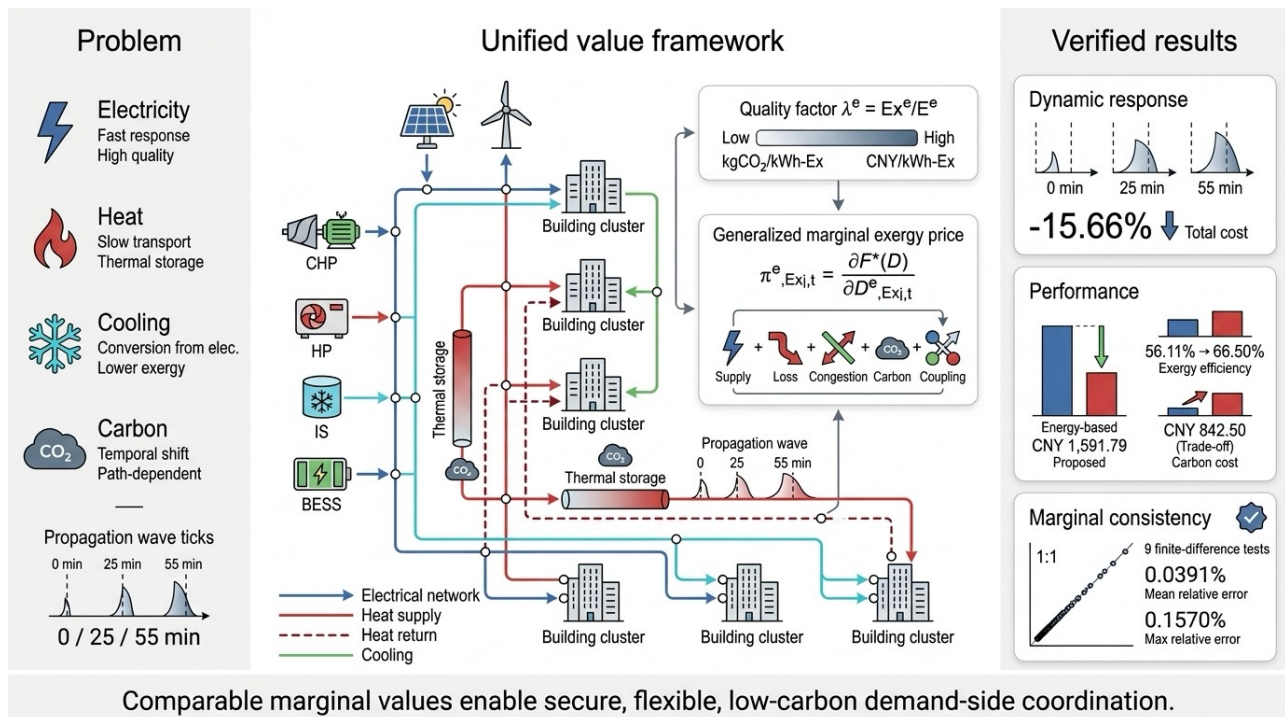
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Received: September 3, 2026; Accepted: September 20, 2026; Published Online: September 24, 2026; <https://doi.org/10.59717/ipj.energy-use.2026.100066>

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GRAPHICAL ABSTRACT



PUBLIC SUMMARY

- Electricity, heat, and cooling differ in transport dynamics, useful-work potential, and associated carbon responsibility; treating them as equivalent kilowatt-hours can therefore distort their system value.
- This study integrates heating-network dynamics, energy quality, and carbon responsibility at the node–time level to derive a comparable value signal for heterogeneous energy carriers.
- In a four-building-cluster case study, the proposed strategy reduces total system cost by 15.66% and raises average exergy efficiency to 66.50%.



INNOVATION
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Energy Use

Energy Use 3 (2026):100066

Contents lists available at INNOVATION PRESS

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Citation: Bu L., Chen X., Yu K., et al. (2026). A Unified Value Signal Driven Strategy for Coordinated Multi-Energy Optimization in Building Clusters. *Energy Use* 2:100066.

China's transition toward an energy powerhouse requires not only a secure and low-carbon energy supply but also flexible and efficient coordination of distributed resources at the demand side. Building-cluster integrated energy systems couple electricity, heat, and cooling, whose transport dynamics, useful-work potential, and carbon responsibilities differ substantially. Conventional energy-based metrics therefore cannot fully capture how network states and energy quality affect nodal marginal values. This study proposes a unified value-signal-driven strategy for coordinated multi-energy optimization in building clusters. First, a coupled energy-carbon flow model represents the hydraulic-thermal dynamics, transport delays, and virtual thermal storage of the heating network, enabling the spatiotemporal tracing of heat and its associated carbon responsibility. Second, exergy is adopted as a common physical basis, while energy quality factors and exergy-carbon potentials quantify carrier-specific useful-work potential and carbon responsibility. An economic and low-carbon dispatch model is then formulated, and the generalized marginal exergy price is defined as the full KKT-based envelope derivative of the optimal value with respect to nodal exergy demand in a continuous convex pricing subproblem. In a four-cluster case study, the load-side response delays obtained from the steady-state, quasi-dynamic, and fully dynamic models are approximately 0, 25, and 55 min, respectively. Relative to unoptimized operation, the proposed strategy reduces daily total cost by 15.66%. Relative to energy-based optimization, it further reduces total cost by CNY 1,591.79 and increases average exergy efficiency from 56.11% to 66.50%, while carbon cost increases by CNY 842.50. Across nine finite-difference tests, the mean and maximum relative errors of the KKT-based marginal values are 0.0391% and 0.1570%, respectively. The framework provides a demand-side mechanism for coordinating local multi-energy resources and supports the development of a more secure, flexible, and low-carbon energy system.

INTRODUCTION

Amid geopolitical uncertainty, volatile international energy markets, and accelerating decarbonization, China's transition toward an energy powerhouse requires a secure, resilient, efficient, and low-carbon energy system. This transition depends not only on diversified and clean energy supply but also on an energy-use revolution that mobilizes distributed generation, storage, and flexible demand at the user side.¹⁻³ Buildings and building clusters accommodate electricity, heating, and cooling loads and provide important interfaces for distributed renewable generation, energy storage, and demand response.⁴⁻⁸ When interconnected through energy networks, multiple buildings can exploit load diversity, share local energy resources, and coordinate storage over a broader spatial scale.⁹ Building clusters are therefore evolving

from passive consumers into active multi-energy resources capable of improving local energy-use efficiency, reducing exposure to external price and supply fluctuations, and providing operational flexibility to the power grid.

A building-cluster integrated energy system (BCIES) combines multiple energy-carrier networks, coupling devices, and storage resources. Combined heat and power (CHP) units, heat pumps (HPs), ice-storage systems (ISs), and electrochemical storage provide cross-carrier conversion and intertemporal flexibility.¹⁰⁻¹³ An operating decision in one energy subsystem can therefore alter the supply-demand balance in another. The underlying networks also exhibit markedly different dynamic behavior. Electricity responds almost instantaneously at the scheduling timescale considered here, whereas heat propagates with the working fluid, and its mass flow, temperature, and stored thermal energy are time dependent. Device capacities, network losses, and transport delays can consequently cause the same load increment to impose different system costs at different nodes and times.^{7,8} Multi-energy coordination must account for both energy balance and the timing, location, and quality of energy delivery. Recent studies have further shown that coordinated building portfolios can function as grid-interactive resources, and that cross-building carbon-electricity-gas trading can activate the flexibility embedded in multi-vector energy systems.¹⁴

Research on integrated energy networks has progressed from steady-state energy-flow analysis to dynamic optimal energy flow, quasi-dynamic scheduling, high-resolution modeling, and dynamic equivalents for heating networks.¹⁵⁻²³ These methods improve the representation of pressure, mass-flow, and temperature propagation and support optimization that accounts for thermal inertia and network-level heat storage. In parallel, carbon-emission-flow tracing methods allocate emission responsibility along network flows and extend conventional source-side accounting to individual nodes and loads.²⁴⁻²⁷ The two research streams, however, remain largely separate. Dynamic network models resolve physical states, whereas carbon-flow models generally allocate responsibility after the energy flows have been determined. In a heating network with substantial thermal storage, heat injected in one period may reach the load in a later period, while its associated carbon responsibility migrates with the in-pipe energy inventory. Without a joint representation, heat supply, load delivery, and carbon attribution can be assigned to inconsistent time periods.

Dynamic network models determine when and in what state energy reaches a node; exergy characterizes how much useful work that energy can deliver. Although electricity, heat, and cooling are all measured in kilowatt-hours, they differ substantially in work potential. Exergy measures the maximum useful work obtainable relative to an environmental dead state and therefore captures both energy quantity and quality.^{28,29} Existing studies have applied exergy analysis to energy hubs, multi-energy networks, device operation, and system planning. Exergy flow, exergy efficiency, and exergy destruc-

tion have been used to identify irreversibility in energy conversion.^{30–36} Exergy-carbon studies further show that energy quality affects the attribution of carbon responsibility among products and conversion pathways.³² In most operational studies, however, exergy metrics remain post-optimization indicators. Energy quality, dynamic network states, and nodal carbon responsibility have not yet been integrated into a common marginal-value formation process.

Marginal value provides a theoretical interface between physical states and operational decisions. A nodal marginal price represents the increase in optimal system cost caused by a local demand change and can allocate boundary energy costs, network losses, and active constraints to individual nodes.³⁷ This principle has been extended from electricity pricing to integrated heat-electricity systems and joint electricity-carbon pricing.^{38,39} Existing methods nevertheless remain predominantly energy based. When 1 kWh of electricity and 1 kWh of low-temperature heat have different work potentials, a value expressed in CNY/kWh lacks a common physical denominator for cross-carrier comparison. Fixed carbon factors also fail to capture heating-network delays, in-pipe thermal storage, and responsibility allocation among CHP products. A key interface is therefore missing: a method for mapping dynamic network states, energy quality, and carbon responsibility into nodal marginal economic quantities defined on a common physical basis.

These gaps form a continuous value-formation chain. Dynamic network states determine the time and condition of energy delivery. Energy quality factors determine the exergy contained in a given amount of energy. Exergy-carbon potentials quantify carbon responsibility per unit of exergy. Device and network constraints determine substitution among energy carriers and flexibility resources. Together, these factors alter the sensitivity of the optimal system value to local demand changes. Accordingly, this study asks: How do dynamic network states and energy quality jointly reshape the nodal marginal values of heterogeneous energy carriers and, in turn, the coordinated operation of devices, networks, and storage in building clusters.

To answer this question, this study proposes a unified value signal-driven strategy for coordinated multi-energy optimization in building clusters. The main contributions are as follows.

A coupled dynamic energy-carbon flow model is developed. The model jointly represents the hydraulic and thermal dynamics of the heating network, transport delays, and virtual thermal storage. It links source-side heat injection, in-pipe energy inventory, and load-side response on a common timescale. Carbon responsibility is updated synchronously with heat transport and storage, thereby capturing the spatiotemporal effect of heating-network dynamics on nodal carbon responsibility.

A mapping is established from a unified physical representation to nodal marginal value. Exergy provides a common basis for heterogeneous energy carriers. Energy quality factors describe quality differences among electricity, heat, and cooling, while exergy-carbon potentials represent carbon responsibility per unit of exergy. Within a continuous convex pricing subproblem, the generalized marginal exergy price is defined as the full envelope derivative of optimal cost with respect to nodal exergy demand, evaluated from the KKT system. The KKT stationarity conditions are then used to decompose this price into supply, loss, congestion, carbon, and coupling components.

A multilevel numerical validation framework is constructed. A case study of four building clusters evaluates dynamic propagation, energy-exergy-carbon-responsibility mapping, coordinated multi-energy dispatch, and marginal-price consistency. Comparisons among unoptimized operation, energy-based optimization, and unified-value-based optimization identify the effects of energy quality and carbon responsibility on operating decisions. Finite-difference re-optimization then verifies the consistency between the KKT-based marginal values and changes in the optimal objective value.

MATERIALS AND METHODS

Building-cluster integrated energy system and problem formulation

This study considers an electricity-heat-cooling integrated energy system comprising four building clusters. The system includes a 9-bus distribution network and a 12-node heating supply-and-return network. Combined heat and power units, heat pumps, ice-storage systems, and energy-storage devices enable conversion among energy carriers and provide intertemporal

flexibility. The external power grid, an external heating network, natural gas, photovoltaic generation, and wind generation constitute the principal energy inputs. Natural gas is used only as the fuel input to the CHP unit. Cooling is produced and balanced locally at each building cluster, and inter-building cooling-network dynamics are not modeled.

Figure 1 presents the physical topology of the system and the unified value-signal analysis framework. The distribution network transfers electricity among the external grid, distributed generators, the CHP unit, and building loads. The heating supply-and-return network connects the CHP unit, the external heating network, and the thermal loads of the four building clusters. Heat transport in the supply–return network is characterized by the evolving mass-flow and temperature states, while the associated carbon responsibility propagates with the thermal flow and in-pipe energy inventory. The HP converts electricity into heat, the IS provides electricity-to-cooling conversion and cold-energy storage, and the battery supplies electricity-side intertemporal flexibility. In addition to these explicit storage devices, the transport and retention of hot water within the pipelines create spatially distributed virtual thermal storage. Multi-energy operation is therefore jointly determined by device conversion, network transport, and intertemporal state evolution.

System operation is represented at three interrelated levels. The first is the dynamic physical-state level. Mass flow, pressure, and temperature in the heating network determine heat-transport velocity, transmission losses, and load-side arrival times. The second is the energy-quality and carbon-responsibility level. Energy quality factors determine the exergy associated with a unit of energy, while energy-flow directions and conversion pathways determine the spatial distribution of carbon responsibility. The third is the operational-decision and marginal-value level. External energy prices, carbon costs, device performance, and network constraints jointly determine the optimal operating point. The sensitivity of the optimal cost to a local exergy-demand change then defines the marginal value at the corresponding node.

Let $\mathcal{T}$ denote the set of scheduling intervals, $\mathcal{E} = \{e, h, c\}$ the set of electricity, heat, and cooling carriers, and $\mathcal{N}$ and $\mathcal{L}$ the sets of network nodes and branches, respectively. Model inputs include electricity, heat, and cooling loads; wind and PV forecasts; time-of-use electricity and heat prices; the natural-gas price; carbon allowances and tiered carbon-price parameters; and device and network parameters. State variables include nodal voltages, pipe pressures and mass flows, supply and return temperatures, storage states, energy-based carbon potentials, and exergy-carbon potentials. The decision variables include electricity and heat purchases, CHP electricity and heat outputs, HP heat output, IS charging and discharging powers, and battery charging and discharging powers. The model outputs comprise the multi-energy dispatch, operating and carbon costs, exergy efficiency, and nodal generalized marginal exergy prices.

The overall calculation follows a sequential chain. Exogenous loads, prices, and device limits define the feasible operating region. The dynamic network model determines the node-time physical states. The exergy-carbon model maps these states into energy-quality and environmental-responsibility attributes. The coordinated optimization model then determines the resource-allocation strategy, after which the optimality conditions map local demand changes into nodal marginal values.

Dynamic energy-carbon flow modeling

Electrical and heating networks exhibit markedly different dynamic characteristics. At the scheduling timescale considered here, the electrical network can be represented using quasi-steady-state power flow. Heat, by contrast, propagates with the working fluid through the pipe network. Variations in mass flow alter heat-transfer capacity, propagation velocity, and the amount of heat stored within the pipelines. A steady-state heating-network model maps source-side heat variations directly to end-use loads, while a quasi-dynamic model that accounts for temperature delay but neglects hydraulic dynamics may underestimate the influence of mass-flow variations.

A hydraulic-thermal model with dynamic flow regulation is therefore developed first. A time-frequency-domain transformation is then applied to obtain the dynamic nodal states. Pipe thermal storage and carbon responsibility are subsequently incorporated into a common temporal recursion, allowing energy states and carbon responsibility to evolve on the same timescale.

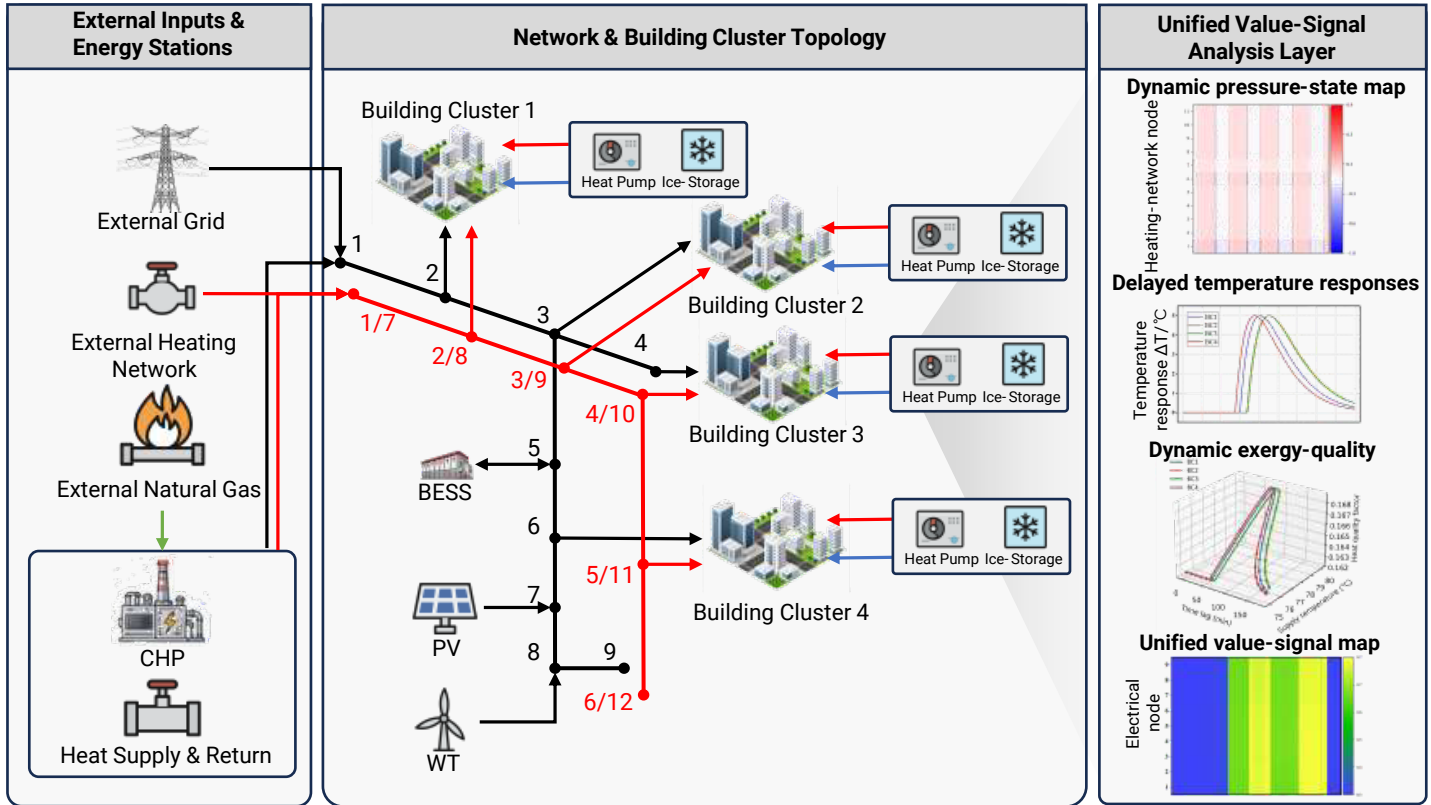


Figure 1. Physical topology of the BCIES and the unified value-signal analysis framework

These dynamic energy-carbon states provide the physical basis for the subsequent exergy characterization and marginal-value analysis.

Hydraulic-thermal dynamic model of the heating network. Consider heating-network pipe k . The pressure and mass flow of the hot water are governed jointly by mass and momentum conservation. To facilitate frequency-domain solution, the nonlinear friction term is linearized to first order around the reference mass flow $G_{0,k}$. The mass-conservation equation for weakly compressible water is written as follows:

$$A_k \frac{\partial p_k}{\partial t} + a_k^2 \frac{\partial G_k}{\partial x} = 0, \quad (1)$$

where A_k is the pipe cross-sectional area, p_k is the internal pressure, G_k is the mass flow, and a_k is the pressure-wave velocity. Eq.1 relates local mass accumulation to pressure variation and flow propagation. Under dynamic flow regulation, pressure and mass flow are treated as spatiotemporal state variables. The momentum-conservation equation is given by:

$$\frac{1}{A_k} \frac{\partial G_k}{\partial t} + \frac{\partial p_k}{\partial x} + \rho g \sin \theta_k + \frac{f_k (2G_{(0,k)} G_k - G_{(0,k)}^2)}{2\rho A_k^2 D_k} = 0, \quad (2)$$

where ρ is the water density, g is the gravitational acceleration, f_k is the friction factor, and D_k is the internal pipe diameter. Elevation changes are small in the case study and are therefore neglected. Eq.2 describes the dynamic balance among fluid inertia, pressure gradient, and frictional resistance. Eq.1 and Eq.2 together determine the time-varying hydraulic state and provide the mass-flow boundary conditions required for thermal propagation.

Heat transport within the pipe is governed by fluid advection and heat dissipation through the pipe wall:

$$c\rho A_k \frac{\partial T_k}{\partial t} + cG_k \frac{\partial T_k}{\partial x} + \lambda_k (T_k - T_a) = 0, \quad (3)$$

where c is the specific heat capacity of water, T_k and T_a are the water and ambient temperatures, respectively, and λ_k is the overall heat-loss coefficient per unit pipe length. The temporal term in Eq.3 represents the variation in the in-pipe thermal inventory, the advective term represents downstream heat transport, and the dissipation term represents heat exchange with the surroundings.

Mass flow therefore affects not only the instantaneous heat-transfer capacity but also the propagation velocity of temperature disturbances and the thermal inventory stored in the pipe. The one-way hydraulic-thermal coupling can be summarized as $G_k(x,t) \rightarrow T_k(x,t) \rightarrow \Phi_k(x,t)$, where the hydraulic state first determines fluid transport and subsequently affects the propagation of temperature and heat flow. This coupling distinguishes the fully dynamic model from a quasi-dynamic formulation in which thermal propagation is evaluated under a prescribed hydraulic state.

Time-frequency transformation and unified two-port representation.

Directly solving the heating-network partial differential equations in the time and space domains requires simultaneous temporal and spatial discretization. The number of state variables and the computational burden increase rapidly with network size and temporal resolution. A Fourier transformation is therefore used to decompose the temporal dynamics into network responses at individual frequencies, which are then solved in the spatial domain.

To formulate the electricity and heating networks within a common mathematical structure, voltage/current, pressure/mass flow, and temperature/heat flow are organized as generalized potential and flow variables. The unified time-domain state equation is written as:

$$A \frac{\partial \chi}{\partial t} + B \frac{\partial \chi}{\partial x} + C\chi = d, \quad (4)$$

$$\chi = [u^T, f^T]^T.$$

Eq.4 does not equate the physical meanings of the different networks. Instead, it provides a common mathematical representation of the relationship between potential and flow variables. Applying the Fourier transformation to the temporal dimension converts time derivatives into frequency-dependent parameters, yielding the frequency-domain constant-coefficient equation:

$$\frac{d}{dx} [\hat{u}; \hat{f}] = [0, Z_\omega; Y_\omega, 0] [\hat{u}; \hat{f}], \quad (5)$$

$$Z_\omega, Y_\omega = F(A, B, C; \omega).$$

For a homogeneous pipe, the network state can be solved independently at each frequency. Let $y_\omega = (Z_\omega Y_\omega)^{1/2}$ and $Z_{c,\omega} = (Z_\omega/Y_\omega)^{1/2}$. A scalar branch, or a

state component obtained after modal decoupling, can then be expressed using the two-port relation between its inlet and outlet states:

$$\begin{aligned} [\hat{u}_k(0, \omega); (\hat{f})_k(0, \omega)] &= T_k(\omega) [\hat{u}_k(l_k, \omega); (\hat{f})_k(l_k, \omega)], \\ T_k(\omega) &= [\cosh(\gamma_\omega l_k), Z_{c,\omega} \sinh(\gamma_\omega l_k); Z_{c,\omega}^{-1} \sinh(\gamma_\omega l_k), \cosh(\gamma_\omega l_k)]. \end{aligned} \quad (6)$$

The two-port model condenses the distributed dynamics within a continuous pipe into an analytical mapping between terminal variables. Different frequency components undergo different amplitude attenuation and phase shifts. The formulation therefore retains the propagation characteristics of network disturbances without requiring dense spatial discretization of each pipeline at every scheduling interval.

The branch-level two-port parameters are assembled into a nodal admittance matrix according to the network topology. After the generalized nodal potential and flow variables have been solved at each frequency, the corresponding time-domain sequences are reconstructed using the inverse discrete Fourier transform:

$$\begin{aligned} \hat{F}(\omega_m) &= Y_{\text{node}} \omega_m \hat{U} \omega_m, \\ U_t &= \frac{1}{M} \sum_{m=0}^{M-1} \hat{U} \omega_m e^{j2\pi m t / M}, \end{aligned} \quad (7)$$

where M is the number of temporal sampling points. The scheduling horizon is 24 h with a 5 min resolution, giving $M=288$. This resolution captures both time-of-use price and device-operation variations and the transport delays within the heating network.

Virtual thermal storage and dynamic carbon conservation. Heating-network dynamics are reflected not only in load-side response delays but also in continuous changes in the in-pipe thermal inventory. When the heat flow entering a pipe exceeds the sum of its outlet heat flow and heat loss, part of the heat is temporarily stored in the pipe. When source output decreases or load demand increases, this stored heat can be released. The pipe network therefore provides distributed and location-dependent virtual thermal storage.

To represent this process, each pipe is divided into elementary segments according to its heat-transport delay. Energy conservation for segment i of pipe k between adjacent time intervals is written as:

$$\frac{H_{k,s,t}^{\text{st}} - H_{k,s,t-1}^{\text{st}}}{\Delta t} = \phi_{k,s,t}^{\text{in}} - \phi_{k,s,t}^{\text{out}} - \phi_{k,s,t}^{\text{loss}} \quad (8)$$

where $H_{k,s,t}^{\text{st}}$ is the stored thermal energy and $\phi_{k,s,t}^{\text{in}}$, $\phi_{k,s,t}^{\text{out}}$, and $\phi_{k,s,t}^{\text{loss}}$ are the inlet heat flow, outlet heat flow, and heat loss, respectively. The segment accumulates heat when its inlet heat flow exceeds the sum of its outlet flow and heat loss; otherwise, it releases previously stored heat. Heat entering the network in one interval may therefore be delivered to end users in a later interval.

The associated carbon responsibility must propagate with both the carrier fluid and the in-pipe energy inventory. The dynamic carbon-conservation equation for an elementary pipe segment is written as:

$$\frac{\rho_{k,s,t}^{\text{out}} H_{k,s,t}^{\text{st}} - \rho_{k,s,t-1}^{\text{out}} H_{k,s,t-1}^{\text{st}}}{\Delta t} = \rho_{k,s,t}^{\text{in}} \phi_{k,s,t}^{\text{in}} - \rho_{k,s,t}^{\text{out}} (\phi_{k,s,t}^{\text{out}} + \phi_{k,s,t}^{\text{loss}}). \quad (9)$$

The elementary segments are connected sequentially along the actual flow direction, with the outlet state of one segment serving as the inlet state of the next. The carbon responsibility generated at a source is therefore transferred progressively to downstream nodes and remains temporally consistent with the corresponding thermal inventory.

The dynamic energy-carbon model thereby distinguishes irreversible heat loss from temporal heat shifting. Heat loss transfers energy to the environment, whereas temporal shifting retains the heat within the network for later delivery. This distinction is required to calculate nodal carbon responsibility and heat-side marginal values consistently.

Exergy-based unified physical characterization of heterogeneous energy carriers

The dynamic energy-carbon flow model provides the energy quantities and associated carbon responsibility at each node and time interval. Electricity, heat, and cooling, however, remain expressed on their respective energy bases. Equal quantities of these carriers do not provide equal amounts of

useful work, and energy conservation alone cannot reveal the quality degradation that occurs when a high-grade carrier is converted into a lower-grade form.

Exergy is therefore adopted as the common physical basis for heterogeneous energy carriers. Exergy represents the maximum useful work obtainable from an energy carrier relative to a specified environmental state and consequently accounts for both energy quantity and quality.¹⁹⁻²⁴ Electricity, heat, and cooling are mapped onto the common unit of kWh-Ex, allowing the useful-work potential retained or destroyed during network transport, device conversion, and end use to be compared directly.

Exergy potential and exergy flow of heat and cooling. The ambient temperature T_a is selected as the common dead-state temperature. When the working-fluid temperature equals the ambient temperature, its useful-work potential relative to the environment is zero. For hot water at temperature T , the specific exergy potential and the heat-exergy flow carried by the mass flow are given by:

$$\begin{aligned} p_T(T) &= c[(T - T_a) - T_a \ln(T/T_a)], \\ \dot{E}_{x_h} &= G[p_T(T^{\text{sup}}) - p_T(T^{\text{ret}})]. \end{aligned} \quad (10)$$

The exergy potential represents the maximum useful work per unit mass relative to the environmental state. The heat-exergy flow is determined jointly by the exergy potential and the mass flow. The dynamic temperature and mass-flow states obtained from the preceding section can therefore be converted directly into dynamic exergy flows at nodes and branches.

Heat transmission is accompanied by temperature reduction, heat dissipation, and flow irreversibility. The inlet exergy flow is consequently generally greater than the outlet exergy flow. Their difference is defined as the pipe exergy destruction and represents the loss of useful-work potential during transmission. A common environmental dead state is used for all sources, networks, devices, and loads to ensure consistency across the system.

Electricity is treated as pure exergy at the operating scale considered, so its exergy is numerically equal to its active electrical energy. Cooling exergy is evaluated relative to the same environmental state using the cooling-supply temperature. The three carriers thus retain their carrier-specific transport and conversion models while being evaluated on a common exergy basis.

Energy quality factor for heterogeneous energy carriers. The energy quality factor is defined as the ratio of exergy to the corresponding energy quantity:

$$\begin{aligned} \lambda_{el} &= 1, \\ \lambda_h &= \frac{p_T(T^{\text{sup}}) - p_T(T^{\text{ret}})}{c(T^{\text{sup}} - T^{\text{ret}})}, \\ \lambda_{(c)} &= \frac{T_a}{T_c} - 1. \end{aligned} \quad (11)$$

The energy quality factor is dimensionless. It is approximated as unity for electricity. For heat, it is determined by the supply, return, and ambient temperatures; for cooling, it depends on the cooling state and the environmental reference. The quality factors of heat and cooling therefore vary with operating conditions.

This state dependence has two implications. First, 1 kWh of heat supplied at different temperatures contains different amounts of exergy. Equal energy quantities consequently need not provide equal useful-work potential. Second, changes in ambient temperature alter the useful-work potential of the same supply state relative to the environmental dead state. The conversion from kWh to kWh-Ex cannot therefore be represented by a fixed empirical coefficient.

At each node and time interval, the energy-to-exergy conversion is expressed as $Ex_{i,t}^e = \lambda_{i,t}^e E_{i,t}^e$, where $E_{i,t}^e$ and $Ex_{i,t}^e$ are the energy and exergy, respectively, of carrier e at node i and interval t , and $\lambda_{i,t}^e$ is the corresponding energy quality factor. This relation maps the dynamic physical state onto a common exergy basis for cross-carrier marginal-value analysis.

Exergy-carbon potential and carbon responsibility allocation. An energy-based carbon potential represents the carbon responsibility carried per unit of energy. Once heterogeneous energy carriers are mapped onto the common exergy basis, their carbon responsibility must also be expressed consistently with energy quality. Exergy-carbon intensity and nodal exergy-

carbon potential are therefore introduced to quantify carbon responsibility per unit of exergy. The conversion and mixing relationships between branch exergy-carbon intensity and nodal exergy-carbon potential are given by:

$$\begin{aligned}\varphi_{j,t}^{Ex} &= \frac{\varphi_{j,t}^{En}}{\lambda_{j,t}}, \\ \rho_{i,t}^{Ex} &= \frac{\sum_{j \in O_i^m} \varphi_{j,t}^{En} Ex_{j,t}}{\sum_{j \in O_i^m} Ex_{j,t}}.\end{aligned}\quad (12)$$

The nodal exergy-carbon potential is measured in kgCO₂/kWh-Ex. It represents the carbon responsibility associated with one unit of exergy at a node. Economic value is subsequently derived from the marginal response of the system cost to nodal exergy demand. For a given energy-based carbon intensity, a lower quality factor results in a higher carbon responsibility per unit of exergy. Exergy-carbon potential therefore enables the environmental attributes of heterogeneous carriers to be compared on a common useful-energy basis.

Exergy destruction during network transport and device conversion does not eliminate the associated carbon responsibility. The responsibility associated with destroyed exergy is retained at the corresponding transport or conversion stage and included in the overall responsibility balance.

A multi-product device such as a CHP unit generates electricity and heat from a common fuel input. Its input carbon responsibility must therefore be allocated among the products. This study allocates that responsibility in proportion to the exergy of each product:

$$\begin{aligned}R_{k,t}^{in} &= \sum_{e \in P_k} R_{k,t}^e + R_{k,t}^{loss} + \Delta R_{k,t}^{st}, \\ R_{k,t}^e &= R_{k,t}^{in} \frac{Ex_{k,t}^e}{\sum_{e \in P_k} Ex_{k,t}^e}.\end{aligned}\quad (13)$$

Eq.13 preserves the total input responsibility while changing its distribution among the products. Relative to energy-based allocation, exergy-based allocation accounts for differences in useful-work potential and assigns responsibility in proportion to each product's exergy contribution.

System, network, and device level exergy indicators. Exergy indicators are defined at the system, network, and device levels to quantify energy-quality utilization:

$$\begin{aligned}r_{sys}^{Ex} &= \frac{\sum_{t,e} Ex_{load,t}^e}{\sum_{t,e} Ex_{ext,t}^e}, \\ r_{net}^{Ex} &= \frac{\sum_{t,k} Ex_{k,t}^{loss}}{\sum_{t,k} Ex_{k,t}^{in}}, \\ r_d^{Ex} &= \frac{\sum_t Ex_{d,t}^{in} - \sum_t Ex_{d,t}^{prod}}{\sum_t Ex_{d,t}^{in}}.\end{aligned}\quad (14)$$

The overall system exergy efficiency is the ratio of useful exergy delivered to end-use demands to total external exergy input. It evaluates system-wide utilization of useful-work potential. The network exergy-destruction rate measures the degradation of useful-work potential during electricity and heat transport. The device exergy-destruction rate quantifies irreversibility within conversion devices such as the CHP unit, HP, and IS.

These indicators reveal energy-quality differences that conventional energy metrics cannot distinguish and support the physical interpretation of alternative dispatch strategies. The operating objective comprises operating and carbon costs. Energy quality affects the feasible region and the optimality conditions through the quality factors, exergy balances, exergy-carbon potentials, and device-conversion constraints. Exergy indicators therefore serve as state and performance measures while indirectly affecting the optimal dispatch and nodal marginal values through the model formulation.

Low-carbon economic multi-energy coordinated optimization model

The coordinated multi-energy optimization model is formulated using the dynamic nodal states and the unified exergy-carbon attributes. The model coordinates external energy purchases, coupling devices, and storage operation while satisfying network, conversion, and intertemporal constraints. Both direct operating costs and the economic cost associated with carbon responsibility are considered.

The dynamic network model defines the attainable physical states. The exergy-carbon model quantifies the energy quality and carbon responsibility

associated with those states. The optimization model then selects the minimum-cost resource combination from the feasible set. The unified value signal is subsequently derived endogenously from the optimality conditions of this optimization problem.

Carbon cost and operating objective. Carbon responsibility is first allocated to loads, products, and loss processes using the exergy-carbon potentials defined above. A tiered carbon-trading mechanism is then used to convert the environmental responsibility into a monetary cost. The actual carbon responsibility, free carbon allowance, and excess responsibility are expressed as:

$$\begin{aligned}R_b^{act} &= \sum_{t,e} \rho_{b,t}^{e,Ex} D_{b,t}^{e,Ex} \Delta t, \\ R_b^{quota} &= \sum_{t,e} a_e D_{b,t}^{e,Ex} \Delta t, \\ \Delta R_b &= R_b^{act} - R_b^{quota}.\end{aligned}\quad (15)$$

The actual system carbon responsibility includes the responsibility assigned to end-use energy consumption and to network and device conversion losses. The free allowance is determined using carrier-specific allowance factors. A surplus allowance has tradable value when the actual responsibility is below the allocated quota, whereas excess responsibility incurs an additional cost. The tiered carbon-trading cost is given by:

$$\begin{aligned}C_{carbon} &= \sum_{s=0, \dots, S} \max p_s (\Delta R_b - x_{b,s}) + y_{b,s}, \\ p_s &= p_0 (1 + sy), \\ x_{b,s} &= sL_b.\end{aligned}\quad (16)$$

The marginal carbon price increases as the excess responsibility moves across successive pricing intervals. The same increase in carbon responsibility may therefore result in different marginal carbon costs at different operating points. This state dependence subsequently appears in the carbon component of the unified value signal. The direct operating cost includes external electricity and heat purchases, natural-gas consumption, and the operating costs of the CHP unit, HP, IS, and storage devices. The overall operating objective is:

$$\begin{aligned}\min F(x) &= C_{OP} + C_{carbon}, \\ C_{OP} &= \sum_t \left[\begin{aligned} &c_t^e P_t^{grid} + c_t^h H_t^{grid} + a_1 P_t^{CHP} + a_2 (P_t^{CHP})^2 \\ &+ b_1 H_t^{CHP} + b_2 (H_t^{CHP})^2 + c_{ph} P_t^{CHP} H_t^{CHP} \\ &+ c_{HP} H_t^{HP} + c_{IS} (Q_t^{ch} + Q_t^{dch}) \end{aligned} \right] \Delta t.\end{aligned}\quad (17)$$

The operating cost represents the direct economic cost of energy procurement and device operation. The carbon cost represents the monetary consequence of carbon responsibility under the prescribed allowance and tiered-pricing rules. Both are expressed in monetary units and are combined in the objective function. Exergy enters through the state representation and constraints rather than through a separate monetized exergy-destruction penalty.

Multi-energy operating constraints. The coordinated optimization must satisfy not only energy balances but also consistency among energy quality, carbon responsibility, and device states. The principal constraints are grouped into exergy balances, exergy-carbon responsibility balances, and device and network operating constraints.

The first group comprises the multi-energy exergy balances. At each node, incoming exergy must balance outgoing exergy, end-use exergy demand, and the associated losses. The electricity, heat, and cooling exergy balances and their corresponding exergy-carbon relationships are represented compactly by:

$$\begin{aligned}\sum_{j \in O_i^m} Ex_{j,t}^e + Ex_{i,t}^{e,src} &= \sum_{j \in O_i^{out}} Ex_{j,t}^e + D_{i,t}^{e,Ex} + Ex_{i,t}^{e,loss}, \\ \sum_{in} \rho^{Ex} Ex &= \sum_{out} \rho^{Ex} Ex + R^{loss} + \Delta R^{st}.\end{aligned}\quad (18)$$

The second group comprises the exergy-carbon responsibility balances. The carbon responsibility carried by nodal input exergy is conserved among output products, network and device losses, and end-use loads. Both carrier conversion and product allocation therefore satisfy an overall responsibility balance.

The third group comprises the conversion, storage, and network constraints. The CHP unit is represented by an electricity-heat feasible oper-

ating region linking fuel input and co-produced outputs. The HP satisfies its electricity-to-heat conversion relationship. The IS and battery satisfy capacity, charging/discharging power, conversion-efficiency, and intertemporal state-recursion constraints. The main device and storage constraints are summarized as:

$$\begin{aligned} \psi_m(P_t^{\text{CHP}}, H_t^{\text{CHP}}) &\leq 0, \\ H_t^{\text{HP}} &= \text{COP}_{\text{HP}} P_t^{\text{HP}}, \\ E_{t+1}^{\text{IS}} &= E_t^{\text{IS}} + \eta^{\text{IS, ch}} Q_t^{\text{ch}} \Delta t - \frac{Q_t^{\text{dis}} \Delta t}{\eta^{\text{IS, dch}}}, \\ E_{t+1}^{\text{B}} &= E_t^{\text{B}} + \eta^{\text{B, ch}} P_t^{\text{ch}} \Delta t - \frac{P_t^{\text{dis}} \Delta t}{\eta^{\text{B, dch}}}. \end{aligned} \quad (19)$$

At the network level, the distribution network satisfies nodal-voltage, branch-power, and capacity limits and is represented using a second-order-cone branch-flow formulation. The pressure, mass-flow, and temperature states of the heating network are determined by Eq.1- Eq.9 and enter the coordinated optimization as state-dependent parameters.

The resulting optimization problem jointly accounts for energy quantity, energy quality, carbon responsibility, and network feasibility. Its optimal solution determines the allocation of external energy and coupling-device outputs, while the optimality information associated with the balance equations and operating limits forms the mathematical basis for the unified value signal.

Derivation and solution of the unified value signal

The coordinated optimization model determines the optimal dispatch under specified loads, prices, and operating limits. The dispatch alone, however, does not quantify the system-wide effect of a local demand change. For example, an incremental heat-exergy demand at a building may be supplied by additional external heat purchases, a change in the CHP heat-to-power ratio, increased HP output, or the release of stored heat. These alternatives entail different network losses, carbon responsibilities, and cross-carrier opportunity costs.

The generalized marginal exergy price is therefore defined from the sensitivity of the optimal value function. Because electricity, heat, and cooling demands have been mapped onto the common kWh-Ex basis, their marginal values can all be expressed in CNY/kWh-Ex. The collection of generalized marginal exergy prices across nodes, time intervals, and energy carriers constitutes the unified value signal.

Lagrangian formulation and full KKT-based envelope derivative. A continuous convex pricing subproblem is constructed around the converged operating state and device mode:⁴⁰

$$\begin{aligned} F^*(D) &= \min_x F(x, D). \\ \text{s.t. } h(x, D) &= 0, \\ g(x, D) &\leq 0, \end{aligned} \quad (20)$$

where x is the vector of continuous decision variables, D is the vector of nodal exergy demands, $h(x, D) = 0$ contains the multi-energy and carbon-responsibility balance constraints, and $g(x, D) \leq 0$ contains network-capacity, device-boundary, and other operating constraints. The corresponding Lagrangian is $\mathcal{L} = F(x, D) + \lambda^T h(x, D) + \mu^T g(x, D)$, where λ and μ are the dual variables associated with the equality and inequality constraints, respectively. Let $P_{i,t}^{e, \text{Ex}}$ denote the exergy-demand power of carrier e at node i and interval t . The corresponding exergy demand over one scheduling interval is $D_{i,t}^{e, \text{Ex}} = P_{i,t}^{e, \text{Ex}} \Delta t$, with units of kWh-Ex. Under an appropriate constraint qualification and strong duality, the optimal solution satisfies primal feasibility, dual feasibility, complementary slackness, and stationarity.

Nodal demand may enter not only a normalized balance equation but also the carbon-responsibility expressions, load-conversion relationships, and other model parameters. Using only the dual variable of a single balance constraint may therefore omit explicit demand-dependent terms. The generalized marginal exergy price is consequently defined using the full KKT-based envelope derivative:

$$\begin{aligned} \pi_{i,t}^{e, \text{Ex}} &= \nabla_{D_{i,t}^{e, \text{Ex}}} F^*(D) = \left[\nabla_{D_{i,t}^{e, \text{Ex}}} F + \mathbf{J}_{h, D_{i,t}^{e, \text{Ex}}}^T \lambda + \mathbf{J}_{g, D_{i,t}^{e, \text{Ex}}}^T \mu \right]^T, \\ \mathbf{J}_{h, D_{i,t}^{e, \text{Ex}}} &= \frac{\partial h}{\partial D_{i,t}^{e, \text{Ex}}}, \mathbf{J}_{g, D_{i,t}^{e, \text{Ex}}} = \frac{\partial g}{\partial D_{i,t}^{e, \text{Ex}}}. \end{aligned} \quad (21)$$

All derivatives in Equation 21 are evaluated at the optimal primal-dual solution. The first term represents the direct dependence of the objective on nodal demand, while the remaining terms account for demand-induced changes in the equality and inequality constraints. The expression reduces to the signed dual variable of a normalized balance equation only when the demand appears exclusively in that equation.

Unified value signal and marginal component decomposition. Equation 21 defines the total marginal system cost associated with a nodal exergy-demand change. To identify its physical origins, the objective and constraint terms in the KKT stationarity conditions are grouped according to their operational roles. The generalized marginal exergy price is then represented as the sum of marginal supply, loss, congestion, carbon, and coupling components:

$$\pi_{i,t}^{e, \text{Ex}} = \pi_{i,t}^{e, \text{sup}} + \pi_{i,t}^{e, \text{loss}} + \pi_{i,t}^{e, \text{cong}} + \pi_{i,t}^{e, \text{carbon}} + \pi_{i,t}^{e, \text{cpl}} \quad (22)$$

The marginal supply component represents the direct incremental cost of external energy procurement or a marginal supply device. The marginal loss component represents the additional network-transport and energy-conversion losses required to serve an incremental end-use demand. The marginal congestion component arises from active line, pipe, and device-capacity constraints. The marginal carbon component captures changes in nodal carbon responsibility and tiered carbon cost. The marginal coupling component represents the opportunity cost transmitted across energy carriers through the CHP unit, HP, IS, and storage devices. The marginal supply component at a reference node is determined directly by the boundary energy price or marginal device cost. The loss, congestion, carbon, and coupling components are propagated to non-reference nodes through the network-sensitivity relationship described below.

For all $i \in \mathcal{N}$, $t \in \mathcal{T}$, and $e \in \mathcal{E}$, the collection of generalized marginal exergy prices is defined as the unified value signal $\Pi^{\text{UVS}} = \{\pi_{i,t}^{e, \text{Ex}} \mid i \in \mathcal{N}, t \in \mathcal{T}, e \in \mathcal{E}\}$. The term unified refers to the common physical denominator of one unit of exergy and to the fact that all marginal quantities are generated by the same multi-energy optimality system. Carrier- and node-specific differences remain because of network position, energy quality, carbon responsibility, and device-coupling conditions.

Value propagation from reference to non-reference nodes. Reference supply nodes and internal load nodes exhibit different marginal-value formation mechanisms. A reference node is directly connected to an external energy source or a principal supply device, and its marginal value is primarily determined by the boundary supply cost, carbon responsibility, and coupling-device state. An internal node additionally incurs the network-loss and capacity opportunity costs associated with delivering energy from the reference node.

The KKT stationarity conditions for the network-state variables establish a sensitivity mapping between marginal values at reference and non-reference nodes. The corresponding coefficient matrix is generally non-square because the numbers of nodal constraints, network-state variables, and marginal components need not coincide. The minimum-norm solution is therefore obtained using the Moore-Penrose generalized inverse:

$$\begin{aligned} \mathbf{A}_i^e &= \mathbf{U}_i^e \Sigma_i^e (\mathbf{V}_i^e)^T, \\ \pi_{i,t}^{e, m} &= (\mathbf{A}_i^e)^{\dagger} \mathbf{r}_{i,t}^{e, m} = \mathbf{V}_i^e (\Sigma_i^e)^{\dagger} (\mathbf{U}_i^e)^T \mathbf{r}_{i,t}^{e, m}, m \in \mathcal{M}. \end{aligned} \quad (23)$$

Eq.23 maps the marginal effects formed at the reference nodes and system boundaries to non-reference nodes through the network-state sensitivity relationship. Matrix rank, condition number, and projection residual are evaluated to ensure numerical stability. When the topology, active constraints, or operating state changes, the price-propagation matrix must be recalculated.

Treatment of nonconvex dispatch and marginal-price consistency. The full multi-energy dispatch model contains piecewise cost functions, charging/discharging states, and other potentially nonconvex operating relationships. The dual variables of the original nonconvex problem cannot in general

be interpreted as globally valid nodal marginal prices. The unified value signal is therefore extracted and validated from a continuous convex pricing subproblem associated with the converged operating mode. The full coordinated dispatch model is first solved to determine the device modes, network states, and dynamic physical parameters. Once consistency between the dynamic states and the dispatch has been achieved, the operating mode is fixed. The branch-flow relations, carbon-cost function, and principal operating limits are then represented in equivalent or locally equivalent continuous convex forms. The resulting pricing subproblem satisfies strong duality, allowing its KKT system to be used to calculate the local marginal values defined in Equation 21.

A finite-difference re-optimization test is used to verify whether the KKT-based marginal quantity accurately represents the change in optimal system cost induced by a local demand perturbation. A small perturbation is applied to a selected nodal exergy demand, and the same pricing subproblem is re-solved:

$$\hat{\pi}_{i,t}^{e,Ex}(\Delta D) = \frac{F^*(D + \Delta D e_{i,t}^e) - F^*(D)}{\Delta D}, \quad (24)$$

$$\varepsilon_{i,t}^e = \left| \frac{\hat{\pi}_{i,t}^{e,Ex} - \pi_{i,t}^{e,Ex}}{\hat{\pi}_{i,t}^{e,Ex}} \right| \times 100\%.$$

The finite-difference marginal value should converge to the full KKT-based envelope derivative as the perturbation decreases. The relative error defined in Equation 24 is used to evaluate the local marginal consistency of the unified value signal.

Joint solution of state, optimization, and value signal. Dynamic network states, energy quality, and device dispatch are mutually coupled. Changes in source and device outputs alter the mass flows and temperatures in the heating network. The updated dynamic states, in turn, change the energy quality factors, exergy destruction, and nodal carbon responsibility. An outer dynamic-state iteration is therefore combined with an inner multi-energy optimization.

Figure 2 summarizes the calculation framework. The dynamic network model first generates the node-time physical states. The exergy-carbon model maps these states into unified energy-quality and carbon-responsibility attributes. The multi-energy optimization then determines the resource-allocation strategy. After the dynamic states and dispatch have converged, the continuous convex pricing subproblem is solved, and the unified value signal is extracted and validated using the KKT system and finite-difference re-optimization.

Algorithm 1 clearly distinguishes between operational optimization and marginal value extraction. Steps 1–7 establish the fixed point between the dynamic network states and the coordinated dispatch. Steps 8–13 then evaluate the marginal-value structure associated with that converged operating point. This sequence ensures that the unified value signal represents the local system cost of an incremental exergy demand under a physically consistent network state. The finite-difference test provides an independent numerical check of this marginal interpretation.

Algorithm 1 Dynamic state-exergy-carbon-optimization-value signal solution

Input: Electric, heat, and cooling load forecasts $D^{e,load}$; PV and wind-power forecasts D^{PV} and D^{WT} ; electricity, heat, and natural-gas prices; carbon quota and ladder carbon-price parameters; network and device parameters; initial hydraulic-thermal states, storage states, and nodal carbon potentials; convergence tolerance ε .

Output: Optimal multi-energy dispatch $\mathbf{v}^*$; converged hydraulic-thermal and storage states; nodal exergy and exergy-carbon states; generalized marginal exergy prices $\pi^{e,Ex}$; unified value signal π^{UVS} ; marginal-price decomposition and finite-difference validation results.

1. Initialize the iteration index $\nu = 0$, hydraulic-thermal states, storage states, and nodal carbon potentials.
2. For the current dispatch $\mathbf{v}^{(k)}$, solve the hydraulic-thermal dynamic model using (1)–(7) to obtain nodal pressures, mass flows, and temperatures over the complete scheduling horizon.
3. Update pipe virtual thermal storage and the associated dynamic carbon inventories using (8) and (9).

4. Calculate exergy flows, energy-quality factors, exergy-carbon potentials, and system/network/device exergy indicators using (10)–(14).
5. Substitute the updated dynamic states and exergy-carbon parameters into the coordinated optimization model defined by (15)–(19), and solve for the new dispatch $\mathbf{v}^{(k+1)}$.
6. Feed $\mathbf{v}^{(k+1)}$ back to the dynamic network model and update the hydraulic-thermal states, thermal-storage states, energy-quality factors, and nodal carbon potentials.
7. Evaluate the changes in the principal state variables and the objective value between two successive iterations. If the convergence criterion is not satisfied, set $\nu = \nu + 1$ and return to Step 2.
8. Fix the converged operating state and device operating mode, and construct the corresponding continuous convex pricing subproblem using (20).
9. Solve the primal-dual pricing subproblem and calculate the complete KKT envelope derivative using (21) to obtain the nodal generalized marginal exergy prices $\pi^{e,Ex}$.
10. Regroup the KKT stationarity terms into supply, loss, congestion, carbon, and coupling contributions, and calculate the corresponding marginal-price components using (22).
11. Map the marginal contributions to non-reference nodes using the network sensitivity relation and the Moore-Penrose generalized inverse in (23).
12. For representative electricity, heat, and cooling demands, perturb $\pi^{e,Ex}$ and re-optimize the pricing subproblem according to (24); calculate the relative error between the finite-difference marginal value and the KKT envelope derivative.
13. Construct the unified value signal $\pi^{UVS} = \{ \pi^{e,Ex} | i \in \mathcal{N} + \mathcal{C} + \mathcal{S} \}$.
14. Return $\mathbf{v}^*$, the converged dynamic energy-carbon-exergy states, π^{UVS} , its marginal components, and the finite-difference validation results.

RESULTS

Test system, data, and comparison cases

The test system comprises a 9-bus distribution network, a 12-node heating supply-and-return network, and four building clusters. The aggregate load shares of Building Clusters 1–4 are 0.3, 0.2, 0.2, and 0.3, respectively. The clusters are connected to electrical buses 2, 3, 4, and 6, and their thermal loads are connected to heating-supply nodes 5, 4, 3, and 2, respectively. The external power grid is connected to bus 1, while the PV array, wind turbine, and CHP unit are connected to buses 7, 8, and 9. The CHP heat source and the external heating network are connected to heating-supply nodes 1 and 6, respectively.

The heating-network supply and return temperatures are set to 80 and 55 °C, respectively, and the ambient reference temperature is 10 °C. The scheduling horizon is 24 h with a 5 min resolution, resulting in 288 consecutive operating intervals. The main device parameters and energy prices are listed in Table S1.

Three cases are considered to isolate the effects of the unified physical characterization. Case U represents the original device operating strategy and serves as the unoptimized baseline. Case E minimizes operating and carbon costs on an energy basis and allocates product carbon responsibility according to energy flow. Case V uses the same loads, energy prices, device capacities, network limits, and economic objective as Case E, but additionally incorporates dynamic exergy states, energy quality factors, exergy-carbon potentials, and exergy-based product-responsibility allocation. The differences between Cases E and V therefore primarily reflect the effects of energy quality and carbon-responsibility representation on operating decisions.

Dynamic response and energy-carbon propagation

Figure 3 compares the load-side responses to the same heat-source disturbance under the steady-state, quasi-dynamic, and fully dynamic hydraulic-thermal models. The comparison is made at heating-supply node 2, which serves Building Cluster 4. The steady-state model responds immediately after the source condition changes. A clear response appears after approximately 25 min in the quasi-dynamic model and after approximately 55 min in the fully dynamic model. Accounting for hydraulic variations therefore introduces an additional delay of approximately 30 min relative to the quasi-dynamic model. This difference shows that mass flow affects not only

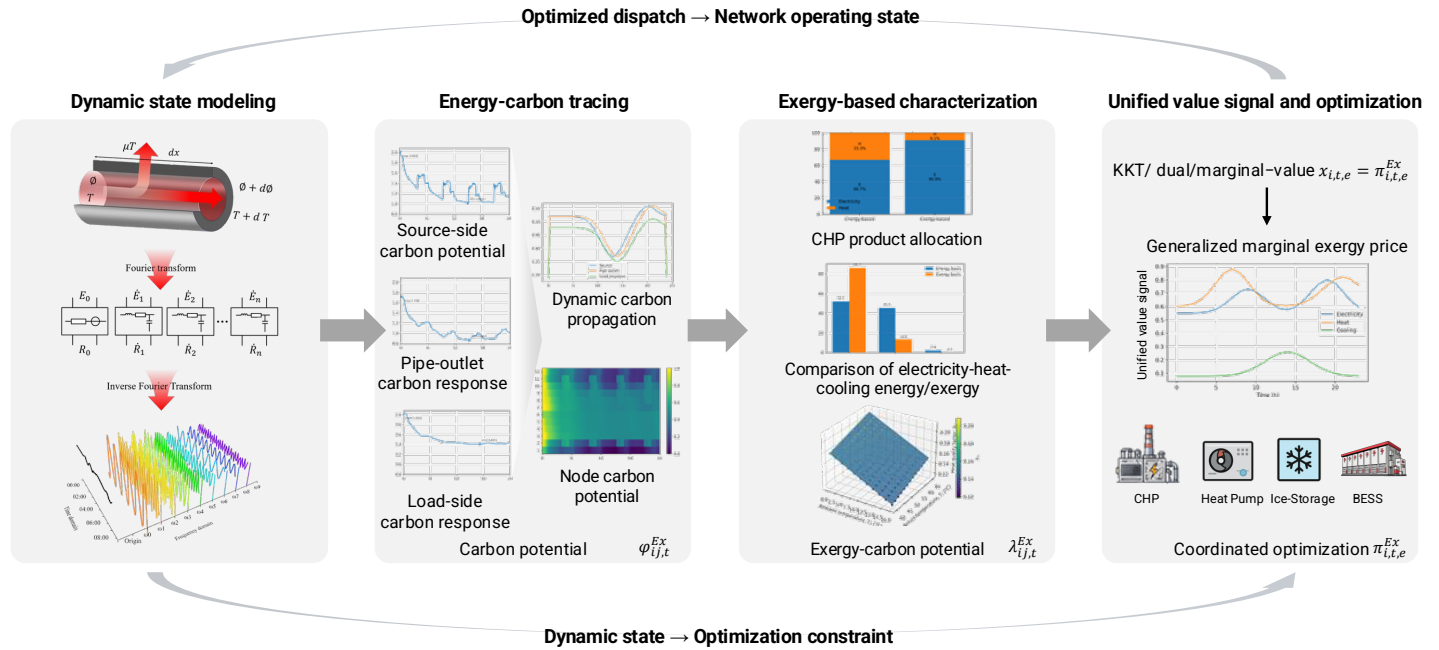


Figure 2. Coupled dynamic-state and value-signal solution framework

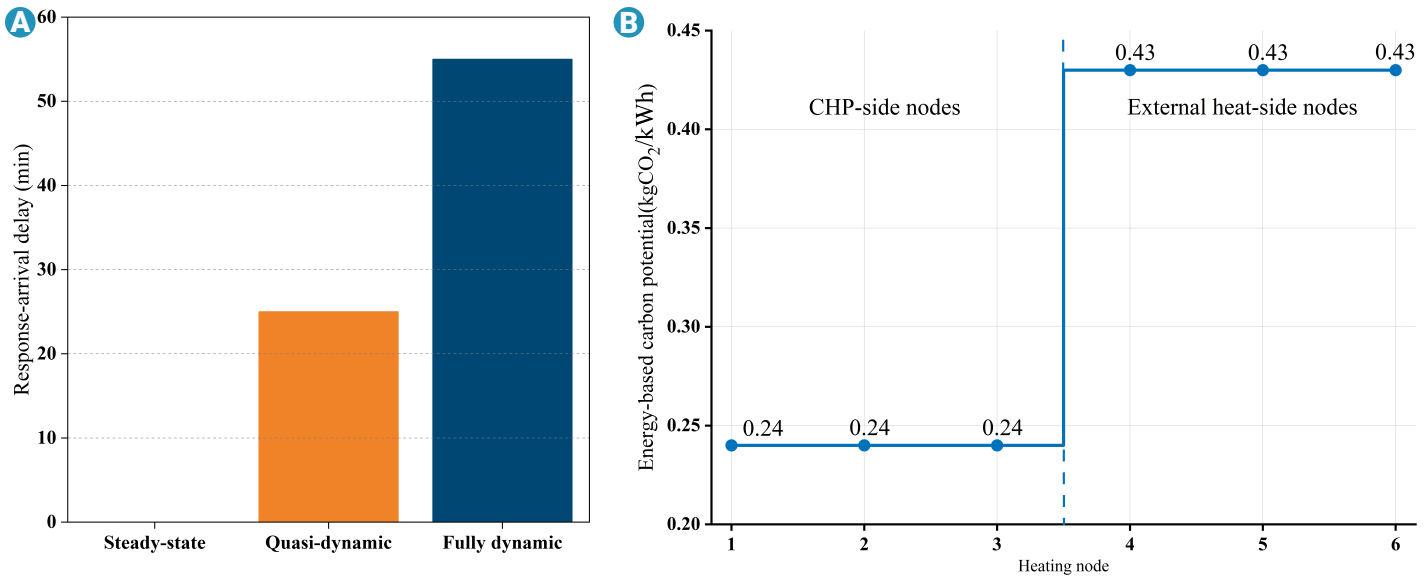


Figure 3. Dynamic response delays and spatial carbon-potential differentiation under different heating-network models

the instantaneous heat-transfer capacity but also temperature propagation by changing the fluid renewal time.

The delay changes the temporal attribution of heat. The steady-state model assigns source-side heat injection and load-side heat delivery to the same interval. The fully dynamic model preserves the intermediate state in which heat has entered the network but has not yet reached the load. Heat injected in advance can first increase the in-pipe thermal inventory and can then be released in subsequent intervals. The heating network therefore provides both spatial heat transport and temporal flexibility through its distributed fluid inventory.

Carbon responsibility exhibits the same propagation behavior. During pipe charging, part of the inlet heat and its associated carbon responsibility is retained within the pipe. During thermal discharge, the accumulated heat and carbon responsibility re-enter the downstream network. The carbon flow associated with thermal storage in supply pipes 2 and 3 changes sign as the pipes alternate between charging and discharging. A positive value indicates that carbon responsibility enters the in-pipe inventory, whereas a negative

value indicates that previously stored responsibility is released with the heat.

Dynamic propagation also creates spatial differences. At 18:00, when multiple heat sources are operating, the energy-based carbon potential is approximately 0.24 kgCO₂/kWh at heating nodes 1–3 near the CHP heat source and approximately 0.43 kgCO₂/kWh at nodes 4–6 near the external heating network. These differences arise from the carbon intensities of the active heat sources, the network-flow directions, and the corresponding transport paths. Thermal carbon responsibility must therefore be evaluated according to node location, source composition, and propagation state. The same thermal demand at different nodes or times can be associated with different upstream supply paths, in-pipe inventories, and marginal carbon costs.

Energy-exergy-carbon responsibility characteristics

The distinction becomes more pronounced at the system level. On an energy basis, electricity, heat, and cooling account for 52.2%, 45.5%, and 2.4% of the total daily useful output, respectively. On an exergy basis, the corre-

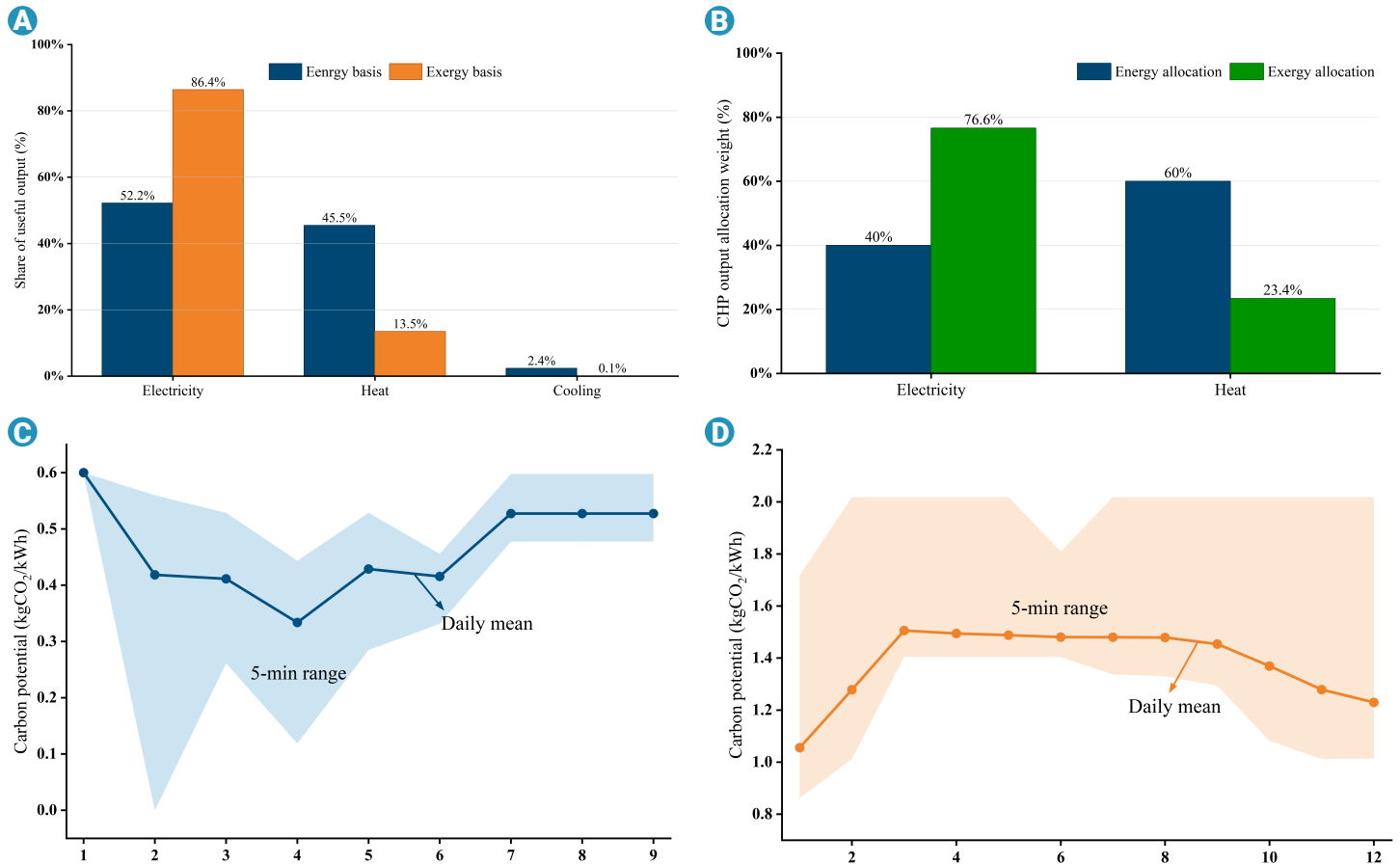


Figure 4. Energy-exergy conversion, carrier structure, CHP product-responsibility allocation, and exergy-performance indicators

sponding shares are 86.4%, 13.5%, and 0.1%. The numerical value of electricity is essentially unchanged because electricity is treated as pure exergy. By contrast, the effective exergy associated with low-temperature heat and cooling decreases substantially. An energy-based representation therefore overstates the contribution of low-grade carriers to the useful output of the system.

Energy quality also changes the allocation of carbon responsibility among CHP products. Based on the actual daily outputs, electricity and heat account for 66.7% and 33.3% of the CHP product energy, respectively. Their exergy shares are 90.9% and 9.1%. For a total input carbon responsibility of 31,989.68 kgCO₂, energy-based allocation assigns 12,795.87 kgCO₂ to electricity and 19,193.81 kgCO₂ to heat. Exergy-based allocation changes these values to 24,498.48 and 7,491.20 kgCO₂, respectively. Both methods preserve the total input responsibility, but the exergy-based method assigns a larger share to electricity because it has a higher useful-work potential.

This redistribution directly affects the subsequent optimization. Electricity and heat are co-produced from a common fuel input, but they have different energy qualities. Changing the allocation basis from energy to exergy therefore changes the carbon costs and marginal contributions assigned to each product. Energy quality consequently affects not only device-performance evaluation but also product-level carbon responsibility and system marginal cost.

Based on the 5 min operating results, the average system exergy efficiency is 66.50%. The average exergy-destruction ratio of the heating network over all 288 intervals is 62.25%, and that of the CHP unit is 54.36%. The HP and IS operate only during selected intervals; their average exergy-destruction ratios over active periods are 37.41% and 94.85%, respectively. The high value for the IS reflects the substantial degradation of useful-work potential when high-grade electricity is converted into low-temperature cooling. The heating-network result indicates that a larger upstream exergy input is required to deliver one unit of useful thermal exergy to an end-use node. These quality losses enter marginal-value formation through the energy

quality factors, device-conversion relationships, and network losses.

Multi-energy coordinated optimization performance

Table S3 compares the operating results for Cases U, E, and V. In the unoptimized Case U, the operating cost, carbon cost, and total cost are CNY 226870.42, CNY 15027.10, and CNY 241897.51, respectively. Energy-based optimization in Case E reduces the total cost to CNY 205602.28, corresponding to a 15.00% reduction relative to Case U. In Case V, the operating and carbon costs are CNY 192072.92 and CNY 11937.57, respectively. The resulting total cost is CNY 204010.49, which is 15.66% lower than that of Case U. The average system exergy efficiency reaches 66.50%.

The comparison between Cases E and V isolates the effect of the unified physical representation. Relative to Case E, Case V reduces the operating cost by CNY 2434.29 but increases the carbon cost by CNY 842.50. The combined effect is a net reduction of CNY 1591.79 in total cost. At the same time, average exergy efficiency increases from 56.11% to 66.50%, an improvement of 10.39 percentage points. Incorporating energy quality therefore changes the relative economics of alternative supply pathways and produces a different balance among operating cost, carbon responsibility, and useful-exergy utilization.

Figure 5 shows the corresponding changes in device dispatch. Relative to Case E, Case V increases daily CHP electricity generation by 1097.74 kWh and reduces grid electricity purchases by 1120.06 kWh. Daily CHP heat output decreases by 5995.53 kWh. The daily HP heat output remains close to 48000 kWh in both cases, indicating that the principal adjustment between Cases E and V occurs through the allocation of CHP electricity and heat outputs and through external energy purchases.

Electricity is treated as pure exergy. Increasing CHP electricity generation therefore displaces high-cost grid purchases while retaining a large share of useful-work potential. Exergy-based allocation assigns a larger share of CHP carbon responsibility to electricity, which raises the carbon cost associated with the electrical product. The reduction in CHP heat output indicates that

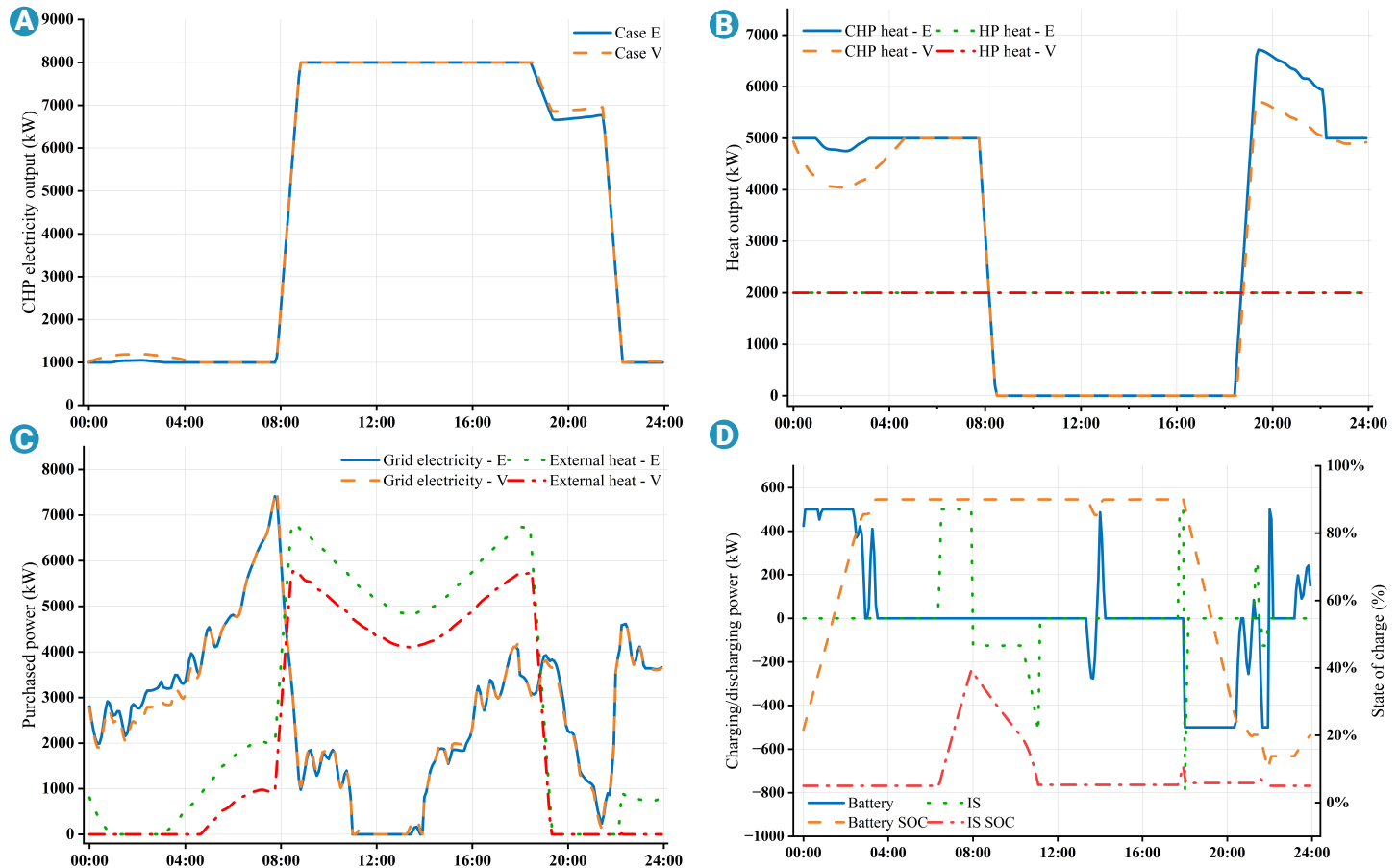


Figure 5. Device-dispatch comparison between the energy-based and unified-value formulations

the model restructures the CHP product mix within its coupled heat-power feasible region rather than simply increasing the total unit output.

Case V achieves the lowest total cost and the highest exergy efficiency, but its carbon cost is slightly higher than that of Case E. This outcome reflects a quantifiable trade-off created by the redistribution of supply pathways and product responsibility. Operating cost, carbon cost, and exergy efficiency represent distinct performance dimensions, and the unified value signal identifies their combined influence on local operating decisions.

Unified value signal and marginal consistency

The unified value signal must satisfy two requirements. First, the marginal values of heterogeneous energy carriers must be comparable on a common physical basis. Second, the marginal values obtained from the KKT system must represent the change in optimal system cost caused by a local demand perturbation. A continuous convex pricing subproblem is therefore constructed at the converged operating point, and representative electricity, heat, and cooling demands are evaluated using both KKT-based marginal analysis and finite-difference re-optimization.

Figure 6A compares the marginal energy prices and generalized marginal exergy prices of three representative energy carriers. At 18:15, the marginal energy price at electrical bus 6 of Building Cluster 4 is 0.7917 CNY/kWh. Because the electricity quality factor is approximately one, the corresponding generalized marginal exergy price is also 0.7917 CNY/kWh-Ex. At 08:45, the marginal energy price at heating node 2 of the same building cluster is 0.6513 CNY/kWh. Under supply, return, and ambient temperatures of 80, 55, and 10 °C, respectively, the heat quality factor is 0.1684. The corresponding generalized marginal exergy price is therefore 3.8669 CNY/kWh-Ex. At 18:25, the marginal energy price of cooling is 0.3113 CNY/kWh. Its quality factor is only 0.02535, resulting in a generalized marginal exergy price of 12.2790 CNY/kWh-Ex.

These results reflect the different demand bases represented by the two prices. Heat and cooling have relatively low prices per unit of energy, but they

also have low energy quality factors. Supplying 1 kWh-Ex of useful heat or cooling therefore requires a larger amount of primary energy than supplying 1 kWh of the corresponding carrier. By expressing all demands on the same useful-work basis, the generalized marginal exergy price enables direct comparison of the marginal values of electricity, heat, and cooling.

The more stringent test is the consistency between the calculated marginal values and the optimal-value sensitivity. Figure 6(b) compares the finite-difference errors obtained using only the nodal balance multiplier with those obtained using the full KKT-based envelope derivative. Across nine perturbation points covering electricity, heat, and cooling demands, the balance-multiplier approximation has a mean relative error of 6.8128% and a maximum error of 13.5164%. After the explicit derivatives associated with carbon responsibility and carrier conversion are included, the mean error decreases to 0.0391%, and the maximum error decreases to 0.1570%.

The error decomposition clarifies the source of this improvement. On the electricity side, the balance multiplier omits an explicit carbon-responsibility contribution of approximately 0.055 CNY/kWh. Cooling demand also enters the relevant constraints through the energy-quality conversion, causing the balance-multiplier approximation to overestimate the marginal value by approximately 13.4%. For heat, most first-order effects are already represented in the associated balance relationships, and the two estimates are therefore close.

The generalized marginal exergy price must consequently be evaluated as the full derivative of the optimal value with respect to nodal exergy demand. Dynamic states and energy quality factors affect both the optimal dispatch and the local sensitivity of the optimal cost. The resulting unified value signal therefore combines a physically consistent demand basis with a validated marginal interpretation.

DISCUSSION

The results show that the unified value signal for heterogeneous energy carriers is jointly shaped by dynamic network states, energy quality, carbon

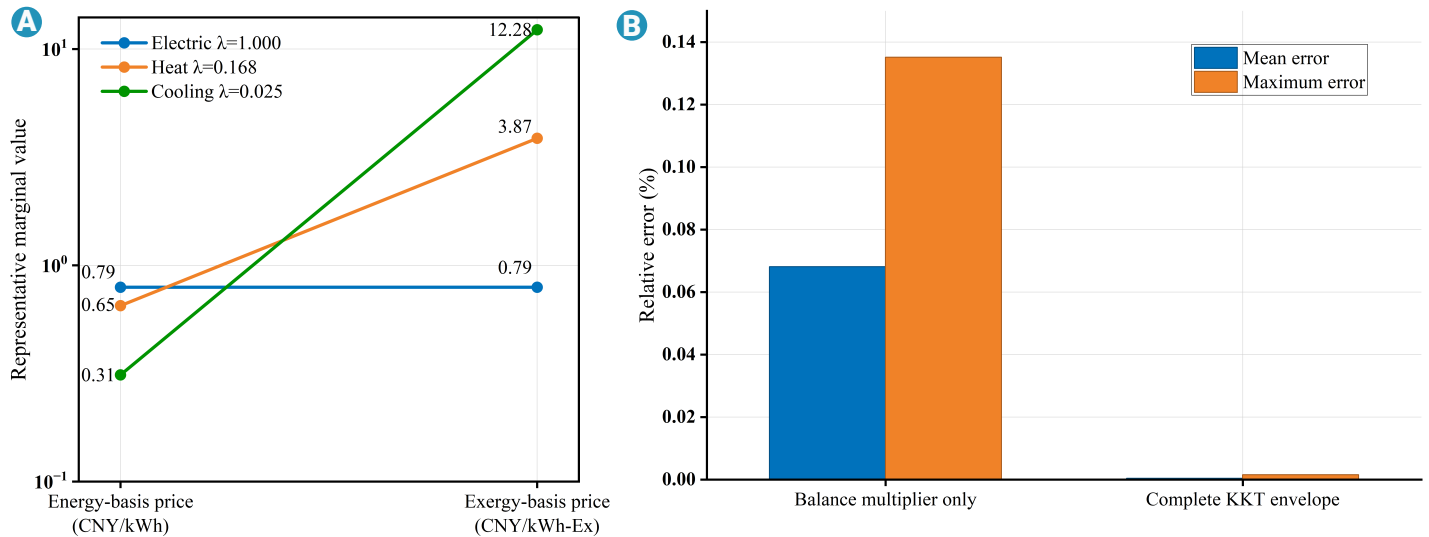


Figure 6. Unified value-signal characteristics and marginal consistency

responsibility, and multi-energy coupling constraints. The dynamic model determines when energy reaches the loads and in what state. Exergy quantifies the useful-work potential of each carrier, while the exergy-carbon potential describes the carbon responsibility associated with one unit of exergy. Once these state-dependent quantities are incorporated into the multi-energy optimization problem, they affect nodal marginal values through the sensitivity of the optimal system cost to local exergy demand. This section discusses how network dynamics, energy quality, and carbon responsibility reshape marginal coordination and what these changes imply for the operation of building clusters.

Dynamic network states reshape nodal marginal value

A defining feature of a heating network is the temporal mismatch between source-side decisions and load-side responses. In the case study, the load-side response delays obtained from the steady-state, quasi-dynamic, and fully dynamic hydraulic-thermal models are approximately 0, 25, and 55 min, respectively. The fully dynamic model introduces an additional delay of approximately 30 min relative to the quasi-dynamic model. This result confirms that variations in mass flow affect heat propagation by changing the fluid renewal time. Consequently, an increase in heat-source output during the current interval may first accumulate as in-pipe thermal storage and may not become available to end users until a later interval.

This intertemporal propagation changes the formation of heat-side marginal value. Under the steady-state formulation, an incremental thermal demand is balanced mainly by increasing the current heat-source output, and its marginal cost therefore remains close to the prevailing energy-purchase or production cost. Once dynamic states are considered, the same demand increment also affects the in-pipe thermal inventory, the heat available for release in subsequent intervals, and the associated network losses. Heat-side marginal value is therefore determined jointly by the current incremental heat requirement, the thermal inventory already available in the network, and the effect of the new demand on future network states. Virtual thermal storage consequently becomes an operational resource with an identifiable opportunity cost.

Carbon responsibility follows the same dynamic transport pathway. During thermal charging, part of the carbon responsibility associated with the incoming heat remains temporarily in the pipe inventory. During thermal discharge, the accumulated responsibility is released to the downstream network together with the stored heat. Different heat sources and transport paths also produce spatial variations. At 18:00, for example, the energy-based carbon potentials at heating-network nodes 1-3 and 4-6 are approximately 0.24 and 0.43 kgCO₂/kWh, respectively. The same 1 kWh thermal demand can therefore carry different carbon responsibilities at different nodes and times, resulting in a clearly time- and location-dependent marginal carbon cost.

These findings indicate that dynamic network modeling changes more

than the numerical resolution of the calculated states. It changes the temporal basis on which nodal marginal values are formed. Steady-state marginal pricing primarily reflects the instantaneous supply-demand balance, whereas a dynamic heating-network model additionally accounts for intertemporal inventories and transport states. Nodal marginal values can therefore reflect the temporal flexibility provided by network thermal inertia and in-pipe storage. This mechanism provides the principal link between dynamic energy-flow modeling and the unified value signal.

Energy quality and carbon responsibility reshape multi-energy coordination

Dynamic states describe the arrival time and condition of energy, whereas exergy characterizes the useful-work potential retained by the delivered energy. Electricity, heat, and cooling account for 52.2%, 45.5%, and 2.4% of the total daily useful output on an energy basis, respectively. On an exergy basis, the corresponding shares become 86.4%, 13.5%, and 0.1%. These changes arise because low-temperature heat and cooling contain only limited exergy relative to the environmental reference state. Energy balance and useful-work-potential balance therefore represent two distinct levels of system performance.

The distinction directly affects the evaluation and dispatch of coupling devices. The HP has a coefficient of performance of 3.5 but an exergy efficiency of approximately 69%. The IS has an energy efficiency ratio of approximately 1.97 but an exergy efficiency of only approximately 11%. Energy-based efficiency describes the amount of output energy obtained from a given input, whereas exergy efficiency identifies the useful-work potential retained during conversion from high-grade electricity to lower-temperature heat or cooling. Coordinated multi-energy operation must therefore consider both the quantity of energy delivered and the exergy retained by each conversion pathway.

The CHP unit further illustrates the interaction between energy quality and carbon responsibility. The energy-based allocation weights of its electricity and heat products are 66.7% and 33.3%, respectively, whereas the corresponding exergy-based weights are 90.9% and 9.1%. With the total input carbon responsibility held constant, exergy-based allocation assigns a larger share to the higher-grade electricity product. Energy quality thus affects not only device-performance assessment but also the carbon costs and marginal contributions attributed to electricity and heat.

This redistribution is reflected in the dispatch differences between Cases E and V. Relative to Case E, Case V increases CHP electricity generation by approximately 1,098 kWh and reduces grid electricity purchases by approximately 1,120 kWh, while decreasing CHP heat production by approximately 5,996 kWh. The optimization does not simply increase or decrease total CHP utilization. Instead, it reallocates electricity and heat production within the coupled feasible operating region of the CHP unit. Electricity provides greater useful-work potential but also assumes a larger share of exergy-based

carbon responsibility. Heat is lower in quality but can be supplied through alternative pathways, including the external heating network, the HP, and virtual thermal storage. The final device schedule results from the interaction among these substitution options.

The full KKT-based envelope derivative further demonstrates that coordinated operation is governed by more than energy-purchase prices. Across the nine finite-difference tests, the mean and maximum relative errors obtained using only the balance-constraint multipliers are 6.8128% and 13.5164%, respectively. After the explicit derivatives associated with carbon responsibility and energy-carrier conversion are included, the errors decrease to 0.0391% and 0.1570%. Energy quality and carbon responsibility therefore enter the marginal response of optimal system cost directly, rather than serving only as post-optimization evaluation metrics.

The unified value signal consequently converts direct supply costs, quality-related losses, carbon responsibility, and cross-carrier opportunity costs into a common marginal quantity. It quantifies the system-wide incremental effect of increasing or reducing one unit of effective demand at a specific node, time, and operating state. It does not impose a fixed ranking among energy carriers; instead, it reveals their state-dependent marginal values under the prevailing network and device conditions.

Trade-off among operating cost, carbon cost, and exergy efficiency

The comparison among operating cases reveals that operating cost, carbon cost, and exergy efficiency do not necessarily improve in the same direction. Relative to the energy-based optimization in Case E, the unified-value formulation in Case V reduces operating cost by CNY 2434.29 and increases carbon cost by CNY 842.50. The net effect is a reduction of CNY 1591.79 in total cost. Over the same comparison, average system exergy efficiency increases from 56.11% to 66.50%, corresponding to an improvement of 10.39 percentage points.

The increase in exergy efficiency primarily indicates better utilization of useful-work potential; it does not directly imply a proportional reduction in carbon responsibility. Case V lowers operating cost by increasing CHP electricity production and reducing purchases of higher-priced grid electricity. At the same time, exergy-based product allocation assigns a larger carbon responsibility to the CHP electricity output. Carbon cost therefore does not decrease in parallel with operating cost. The net economic benefit arises because the reduction in operating cost exceeds the increase in carbon cost.

This non-monotonic relationship is important for multi-energy optimization because the three performance indicators arise from different physical and economic mechanisms. Operating cost is governed mainly by energy prices and device operating expenses. Exergy efficiency measures the proportion of input useful-work potential ultimately delivered to effective demand. Carbon cost depends on carbon responsibility, free allowances, and the active tier of the carbon-pricing function. A change in the energy-supply structure can therefore improve one indicator while producing a different response in another.

At the broader energy-system level, the proposed framework translates the strategic objective of building an energy powerhouse into an operational demand-side mechanism. By coordinating distributed renewable generation, CHP, heat pumps, electrical and thermal storage, and external energy interfaces through a common value signal, building clusters can function as controllable multi-energy resources rather than passive loads. Such coordination can improve the utilization of local clean energy, reduce exposure to external energy-price and supply fluctuations, and provide flexible support to the power grid. These functions are consistent with the emerging role of urban energy use in grid interaction and with the broader transition from isolated building optimization to coordinated building-to-grid operation.^{1,3,4,9,10,14}

CONCLUSIONS

This study developed a unified value-signal-driven strategy for coordinated multi-energy optimization in building clusters. The framework links hydraulic-thermal dynamics, virtual thermal storage, energy quality, carbon responsibility, and nodal marginal value within a common calculation chain. Exergy provides a shared physical basis for electricity, heat, and cooling, while the full KKT-based envelope derivative converts state-dependent phys-

ical and carbon attributes into generalized marginal exergy prices.

The fully dynamic heating-network model produces a load-side response delay of approximately 55 min, compared with 25 min under the quasi-dynamic formulation and an instantaneous response under the steady-state formulation. The energy-to-exergy conversion changes the electricity, heat, and cooling shares from 52.2%, 45.5%, and 2.4% to 86.4%, 13.5%, and 0.1%, respectively. Relative to energy-based optimization, the unified-value formulation reduces total cost by CNY 1,591.79 and increases average exergy efficiency from 56.11% to 66.50%, while carbon cost increases by CNY 842.50.

Across nine perturbation tests, the mean and maximum errors of the balance-multiplier approximation are 6.8128% and 13.5164%, whereas those of the full KKT-based envelope derivative are only 0.0391% and 0.1570%. These results confirm that explicit demand effects associated with carbon responsibility and carrier conversion must be included in nodal marginal-value calculations. At scale, the proposed value signal can support the aggregation of building clusters into dispatchable demand-side resources, providing a technical basis for improved energy-use efficiency, renewable-energy integration, and operational resilience in the transition toward a secure, low-carbon, and intelligent energy system.

REFERENCES

- Chen X., Liu J., Lin B., et al. (2025). Promote the Revolution in Energy Use to Enhance the Impetus for Green Transformation and Development. *Energy Use* 1:100001. DOI:10.59717/ipj.energy-use.2025.100001
- Chen X. (2024). Green and low-carbon energy-use. *Innov. Energy* 1:100003. DOI:10.59717/j.xinn-energy.2024.100003
- Chen X. (2025). Urban Energy Use and Its Friendly Interaction with Power Grid. *Energy Use* 1:100011. DOI:10.59717/ipj.energy-use.2025.100011
- Xiang Y., Chen L., Wuyun Q., et al. (2025). A Comprehensive Review of Building-to-Grid Interaction. *Energy Use* 2:100043. DOI:10.59717/ipj.energy-use.2025.100043
- Ma L., Liu N., Wang L., et al. (2026). Multi-party energy management for smart building cluster with PV systems using automatic demand response. *Energy Build.* 121:11–21. DOI:10.1016/j.enbuild.2016.03.072
- Dawes G., Kirant-Mitić T., Jiang Z., et al. (2025). Energy flexibility at multi-building scales: A review of the dominant factors and their uncertainties. *Energy Build.* 346:116157. DOI:10.1016/j.enbuild.2025.116157
- Jing Z., Chen X., Bu L., et al. (2024). Intelligent optimization for building energy management considering indoor heat transfer. *Innov. Energy* 1:100058. DOI:10.59717/j.xinn-energy.2024.100058
- Li S., Chen X., Bu L., et al. (2024). Two-stage optimization for the air conditioning system in public buildings with flexible control of indoor load. *Energy Build.* 312:114162. DOI:10.1016/j.enbuild.2024.114162
- Zhu J., Han L., Yu J., et al. (2026). Planning High-PV Penetration Park: Enhancing Local PV Utilization and Economic Efficiency through Multi-Energy Storage. *Energy Use* 2:100042. DOI:10.59717/ipj.energy-use.2025.100042
- Zhu D., Korolija I., Dong Z.Y., et al. (2025). Coordinated Optimization of Carbon-Electricity-Gas Trading in Multi-Vector Energy Systems across Buildings. *Energy Use* 1:100003. DOI:10.59717/ipj.energy-use.2025.100003
- Ghilardi L.M.P., Castelli A.F., Moretti L., et al. (2021). Co-optimization of multi-energy system operation, district heating/cooling network and thermal comfort management for buildings. *Appl. Energy* 302:117480. DOI:10.1016/j.apenergy.2021.117480
- Qian C., He N., Cheng Z., et al. (2024). Optimal scheduling of building energy system with integrated virtual energy storage based on multi-task model predictive control. *J. Build. Eng.* 98:111185. DOI:10.1016/j.jobe.2024.111185
- Zhang H., Xiao F., Zhang C., et al. (2023). A multi-agent system based coordinated multi-objective optimal load scheduling strategy using marginal emission factors for building cluster demand response. *Energy Build.* 281:112765. DOI:10.1016/j.enbuild.2022.112765
- Hua H., Chen X., Gan L., et al. (2024). Demand-side joint electricity and carbon trading mechanism. *IEEE Trans. Ind. Cyber-Phys. Syst.* 2:14–25. DOI:10.1109/TICPS.2023.3335328
- Huang W., Zhang N., Cheng Y., et al. (2020). Multienergy networks analytics: Standardized modeling, optimization, and low carbon analysis. *Proc. IEEE* 108:1411–1436. DOI:10.1109/JPROC.2020.2993787
- Yao S., Gu W., Lu S., et al. (2021). Dynamic optimal energy flow in the heat and electricity integrated energy system. *IEEE Trans. Sustain. Energy* 12:179–190. DOI:10.1109/TSTE.2020.2988682
- Chen X., Bu L., Chen C., et al. (2024). An operational optimization method of regional multi-energy system considering thermal quasi-dynamic characteristics. *CSEE J. Power Energy Syst.* 10:2551–2563. DOI:10.17775/CSEEJPES.2020.03830
- Lu S., Gu W., Zhou S., et al. (2020). High-resolution modeling and decentralized dispatch of heat and electricity integrated energy system. *IEEE Trans. Sustain. Energy* 11:1451–1463. DOI:10.1109/TSTE.2019.2927637
- Yang J., Zhang N., Botterud A., et al. (2020). On an equivalent representation of the

- dynamics in district heating networks for combined electricity-heat operation. *IEEE Trans. Power Syst.* **35**:560–570. DOI:[10.1109/TPWRS.2019.2935748](https://doi.org/10.1109/TPWRS.2019.2935748)
20. Chen B., Wu W., Guo Q., et al. (2022). An efficient optimal energy flow model for integrated energy systems based on energy circuit modeling in the frequency domain. *Appl. Energy* **326**:119923. DOI:[10.1016/j.apenergy.2022.119923](https://doi.org/10.1016/j.apenergy.2022.119923)
 21. Chen X., Hu Y., Zhao J., et al. (2025). Downscaling deconstruction, hybrid semi-mechanism state estimation and cascaded dynamic equivalent modelling of complex district heating networks. *Energy* **322**:135678. DOI:[10.1016/j.energy.2025.135678](https://doi.org/10.1016/j.energy.2025.135678)
 22. Wang X., Han L., Wang C., et al. (2023). A time-scale adaptive dispatching strategy considering the matching of time characteristics and dispatching periods of the integrated energy system. *Energy* **267**:126584. DOI:[10.1016/j.energy.2022.126584](https://doi.org/10.1016/j.energy.2022.126584)
 23. Qin Y., Liu P. and Li Z. (2022). Multi-timescale hierarchical scheduling of an integrated energy system considering system inertia. *Renew. Sustain. Energy Rev.* **169**:112911. DOI:[10.1016/j.rser.2022.112911](https://doi.org/10.1016/j.rser.2022.112911)
 24. Cheng Y., Zhang N., Wang Y., et al. (2019). Modeling carbon emission flow in multiple energy systems. *IEEE Trans. Smart Grid* **10**:3562–3574. DOI:[10.1109/TSG.2018.2830775](https://doi.org/10.1109/TSG.2018.2830775)
 25. Kang C., Zhou T., Chen Q., et al. (2015). Carbon emission flow from generation to demand: A network-based model. *IEEE Trans. Smart Grid* **6**:2386–2394. DOI:[10.1109/TSG.2015.2388695](https://doi.org/10.1109/TSG.2015.2388695)
 26. Huang J., Duan W., Zhou Q., et al. (2022). Methodology for carbon emission flow calculation of integrated energy systems. *Energy Rep.* **8**:1090–1097. DOI:[10.1016/j.egy.2022.05.263](https://doi.org/10.1016/j.egy.2022.05.263)
 27. Bu L., Chen X., Gan L., et al. (2023). Low-carbon operation method of the building based on dynamic carbon emission factor of power system. *IET Smart Grid* **6**:67–85. DOI:[10.1049/stg2.12085](https://doi.org/10.1049/stg2.12085)
 28. Tsatsaronis G. (2007). Definitions and nomenclature in exergy analysis and exergoeconomics. *Energy* **32**:249–253. DOI:[10.1016/j.energy.2006.07.002](https://doi.org/10.1016/j.energy.2006.07.002)
 29. Rosen M.A., Dincer I. and Kanoglu M. (2008). Role of exergy in increasing efficiency and sustainability and reducing environmental impact. *Energy Policy* **36**:128–137. DOI:[10.1016/j.enpol.2007.09.006](https://doi.org/10.1016/j.enpol.2007.09.006)
 30. Tahir M.F., Chen H. and Han G. (2021). Exergy hub based modelling and performance evaluation of integrated energy system. *J. Energy Storage* **41**:102912. DOI:[10.1016/j.est.2021.102912](https://doi.org/10.1016/j.est.2021.102912)
 31. Li J., Wang D., Jia H., et al. (2022). Mechanism analysis and unified calculation model of exergy flow distribution in regional integrated energy system. *Appl. Energy* **324**:119725. DOI:[10.1016/j.apenergy.2022.119725](https://doi.org/10.1016/j.apenergy.2022.119725)
 32. Yang W., Huang Y., Zhang T., et al. (2023). Mechanism and analytical methods for carbon emission-exergy flow distribution in heat-electricity integrated energy system. *Appl. Energy* **352**:121980. DOI:[10.1016/j.apenergy.2023.121980](https://doi.org/10.1016/j.apenergy.2023.121980)
 33. Chen J., Chen H., Liang Z., et al. (2024). A joint solution framework for the optimal operation and evaluation of integrated energy systems based on exergy. *Energy Convers. Manag.* **301**:118071. DOI:[10.1016/j.enconman.2024.118071](https://doi.org/10.1016/j.enconman.2024.118071)
 34. Chen J., Chen H., Lin Z., et al. (2023). Multi-objective optimal dispatch of integrated heat and electricity energy systems considering heat load energy quality. *Energy Rep.* **9**:1191–1200. DOI:[10.1016/j.egy.2023.05.081](https://doi.org/10.1016/j.egy.2023.05.081)
 35. Liu S., Han S., Tian J., et al. (2024). A multi-objective optimization scheduling approach of integrated energy system considering the exergy efficiency using the variable step-size approximation method. *Energy* **311**:133370. DOI:[10.1016/j.energy.2024.133370](https://doi.org/10.1016/j.energy.2024.133370)
 36. Peng S., Liu S. and Wu X. (2025). Optimal scheduling of integrated energy system considering exergoeconomic performance. *Energy* **320**:135171. DOI:[10.1016/j.energy.2025.135171](https://doi.org/10.1016/j.energy.2025.135171)
 37. Chen L., Suzuki H., Wachi T., et al. (2002). Components of nodal prices for electric power systems. *IEEE Trans. Power Syst.* **17**:41–49. DOI:[10.1109/59.982191](https://doi.org/10.1109/59.982191)
 38. Deng L., Li Z., Sun H., et al. (2019). Generalized locational marginal pricing in a heat-and-electricity-integrated market. *IEEE Trans. Smart Grid* **10**:6414–6425. DOI:[10.1109/TSG.2019.2904522](https://doi.org/10.1109/TSG.2019.2904522)
 39. Li J., Xiang Y., Gu C., et al. (2023). Formulation of locational marginal electricity-carbon price in power systems. *CSEE J. Power Energy Syst.* **9**:1968–1972. DOI:[10.17775/CSEEJPES.2021.06400](https://doi.org/10.17775/CSEEJPES.2021.06400)
 40. Jiang Y., Wan C., Botterud A., et al. (2021). Convex relaxation of combined heat and power dispatch. *IEEE Trans. Power Syst.* **36**:1442–1458. DOI:[10.1109/TPWRS.2020.3025070](https://doi.org/10.1109/TPWRS.2020.3025070)

FUNDING AND ACKNOWLEDGMENTS

This work was supported by National Natural Science Foundation of China (No. U22B20112)

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DECLARATION OF INTERESTS

Xingying Chen is the Editor-in-Chief of Energy Use and was recused from reviewing or making final decisions on this manuscript. Peer review was handled independently of this member and their research group.

ETHICAL STATEMENT AND PATIENT CONSENT

Not Applicable.

DATA AND CODE AVAILABILITY

Not Applicable

SUPPLEMENTAL INFORMATION

It can be found online at <https://doi.org/10.59717/ipj.energy-use.2026.100066>