

Observation of higher-order topological bound states in the continuum using ultracold atoms

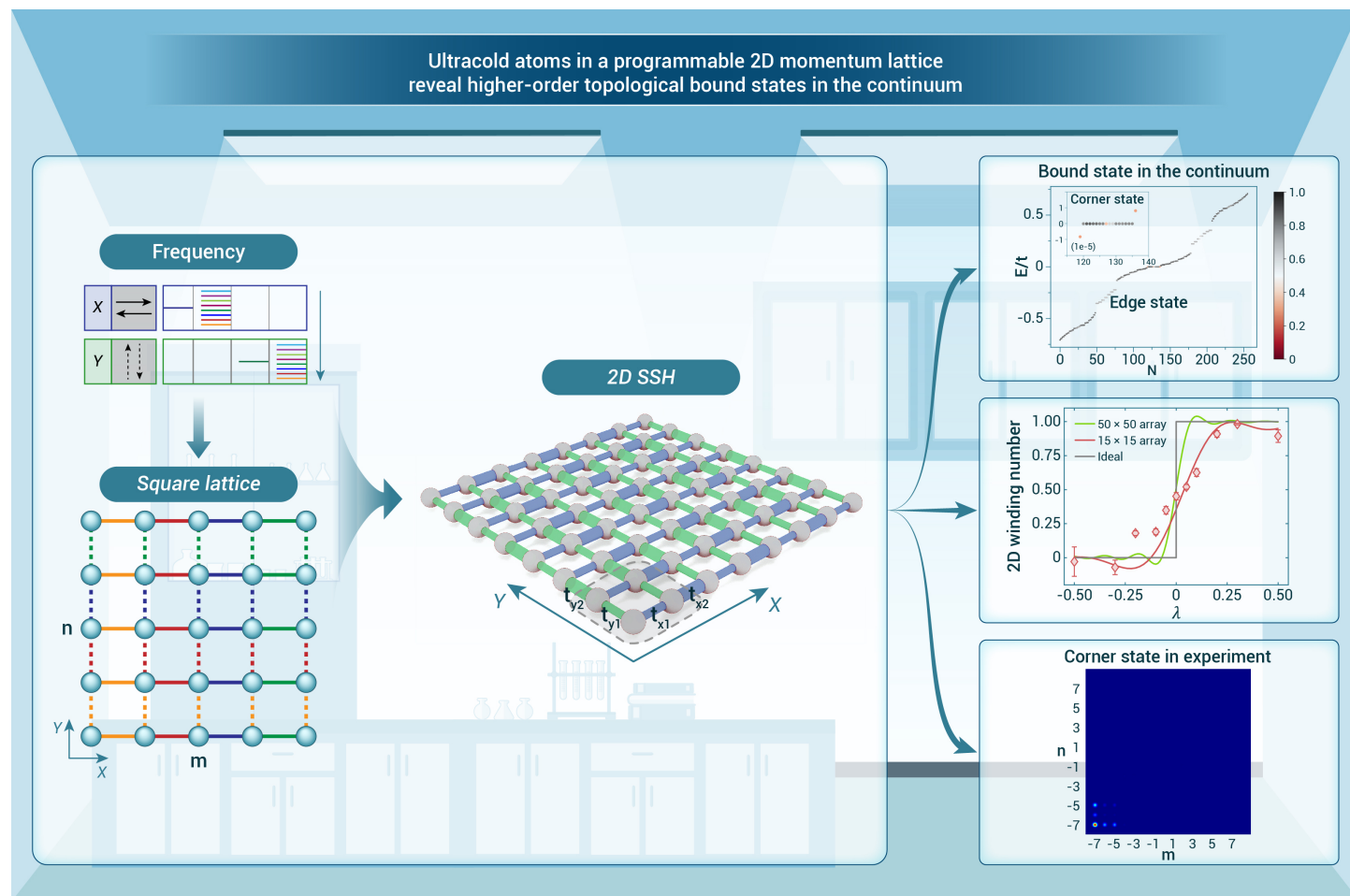
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GRAPHICAL ABSTRACT



PUBLIC SUMMARY

- A two-dimensional momentum lattice is engineered using ultracold atoms.
- Corner and edge bound-state dynamics reveal higher-order topology.
- The corner state is identified as a higher-order topological bound state in the continuum.
- Adiabatic preparation and topological transition are probed in a programmable cold-atom platform.

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Simulating higher-order topological materials in synthetic quantum matter is an active research frontier for its theoretical significance in fundamental physics and promising applications in quantum technologies. Here we experimentally implement two-dimensional (2D) momentum lattices with highly programmable ability using ultracold ⁸⁷Rb atoms. Through precise control of experimental parameters, we simulate a 2D Su-Schrieffer-Heeger model with this technique, and observe the characteristic dynamics of corner and edge-bound states, where the corner state is identified as a higher-order topological bound state in the continuum. We further study the adiabatic preparation of the corner state by engineering evolutions with time-dependent Hamiltonians. We also demonstrate the higher-order topological phase transition by measuring both the bulk topological invariant and the topological corner state. Our new platform opens the avenue for exploring the exotic dynamics and topology in higher synthetic dimensions, making use of the rich degrees of freedom of cold atoms systems.

INTRODUCTION

Topological materials have attracted considerable interest due to their importance in fundamental physics and potential applications in quantum devices.^{1–6} Recently, a new class of topological materials, dubbed higher-order topological insulators (HOTIs),^{7–15} has been proposed and verified on various physical platforms such as microwave resonators,¹⁶ photonics,^{17–23} acoustics,^{24–32} electric circuits,^{33,34} and solid materials.³⁵ Generally, a d -dimensional lattice can host $(d-n)$ -dimensional states at its boundaries, protected by the crystalline symmetries, and referred to as n th-order topological insulators. For the most commonly studied HOTIs,^{7–10} their characterization relies on the boundary states being embedded within the band gap, and distinctly isolated from other states. Whereas for some HOTIs,^{36,37,38} the corresponding boundary states are embedded within the bulk continuum yet still localized. These states are known as the bound states in the continuum (BICs).

The BICs maintain the properties of boundary states while remaining degenerate with states in the bulk bands. Meanwhile, they can still be easily excited and detected without hybridizing with the bulk states, and can even acquire topological protection to become higher-order topological BICs.^{38,39} In recent years, higher-order topological BICs have been extensively investigated across diverse systems, including the acoustic system,³⁷ photonic waveguides^{39,40,41} and topological electric circuits.⁴² However, the demonstration of the higher-order BICs in ultracold atoms remains elusive, wherein the multitude of highly controllable degrees of freedom offer unprecedented opportunities in manipulating the higher-order topological quantum matter.^{43–46}

In this work, we report the experimental observation of HOTI and the associated higher-order topological BICs in ultracold ⁸⁷Rb atoms. This is achieved through our newly developed momentum-lattice platform, where 2D lattice Hamiltonians can be engineered in a programmable fashion in the momentum space of cold atoms. Thanks to the flexible control over the lattice geometry and site-resolved hopping patterns and phases, momentum lattices based on ultracold atoms have unveiled a wealth of quantum dynamic phenomena over the past decade, including exotic topological matter and transport,^{47–50} non-Hermitian quantum control,^{51,52} interplay of

mobility edges and interaction,^{53,54,55} as well as correlated dynamics in frustrated geometries.^{56,57,58} While all these studies focus on one-dimensional or quasi-two-dimensional lattices, extending the momentum-lattice technique to higher dimensions is a highly desired yet challenging goal. Our implementation of the 2D square momentum lattice is hence a significant step toward achieving this goal.

Building on our latest technical advancements, we implement the 2D Su-Schrieffer-Heeger (SSH) model and observe the characteristic dynamics of both corner and edge bound states. Crucially, the corner state is identified as a higher-order topological BIC within this model. Enabled by the unique programmable control of our new platform, we experimentally confirm this by studying the adiabatic preparation of the corner state. By analyzing the conditions required for successful adiabatic preparation, our results provide both theoretical insights and practical pathways for accessing topological BICs. Furthermore, we directly probe the higher-order topological transition of the system through measuring the bulk topological invariant and tracking the emergence of the BICs. These findings open new avenues for exploring BICs and higher-order topology in cold-atom systems, where the tunability inherent to their many-body environment facilitates future studies of exotic topological phases.

MATERIALS AND METHODS

Implementing two-dimensional momentum lattice

We begin with a brief summary on the engineering of the 2D momentum lattice. Building upon the state-of-the-art 1D momentum-lattice technique, we created a 2D lattice by applying two sets of perpendicular Bragg lasers on a ⁸⁷Rb Bose-Einstein condensate (BEC) with 4×10^4 atoms,^{59–62} as shown in Figure 1A. The lattice lasers separately couple discrete momentum states of $|F=1, mF=-1\rangle$ in ground-state hyperfine manifold along the x and y directions, giving rise to a 2D square lattice configuration. The discrete momentum states of this 2D square lattice are denoted as $p = (2m\hbar k_x + 2n\hbar k_y)$, where the wave vectors $k_{x,y} = 2\pi/\lambda_{x,y}$ (with $\lambda_{x,y} = 794.7\text{nm}$), and $m, n \in \mathbb{Z}$ label the synthetic lattice sites (see Figure 1B). Hoppings between neighbouring momentum states along the x or y direction are realized by the corresponding two-photon Bragg processes, enabling a programmable control over the tunnelling terms of the synthetic 2D tight-binding model. Under our scheme, the nearest-neighbor couplings within the same column or row are induced by the same pair of two-photon Bragg process and therefore have the same coupling rates (see Figure 1B). To resolve the atomic population of the 2D momentum-lattice sites, we apply another probe laser from the top to perform the time-of-flight absorption imaging. More experimental details can be found in the Supplementary Material.

Based on the aforementioned experimental platform, we construct the 2D SSH model, as illustrated in Figure 2A. The model exhibits C_{4v} symmetry, and each unit cell contains four sublattice sites. The implemented 2D momentum lattice is described by the tight-binding Hamiltonian

$$H_{\text{eff}} = -\sum_n \left(\sum_{m \in \text{odd}} t_1 c_{m+1,n}^\dagger c_{m,n} + \sum_{m \in \text{even}} t_2 c_{m+1,n}^\dagger c_{m,n} \right) - \sum_m \left(\sum_{n \in \text{odd}} t_1 c_{m,n+1}^\dagger c_{m,n} + \sum_{n \in \text{even}} t_2 c_{m,n+1}^\dagger c_{m,n} \right) + \text{H.c.} \quad (1)$$

where $c_{m,n}^\dagger$ ($c_{m,n}$) is the creation (annihilation) operator for atoms on lattice

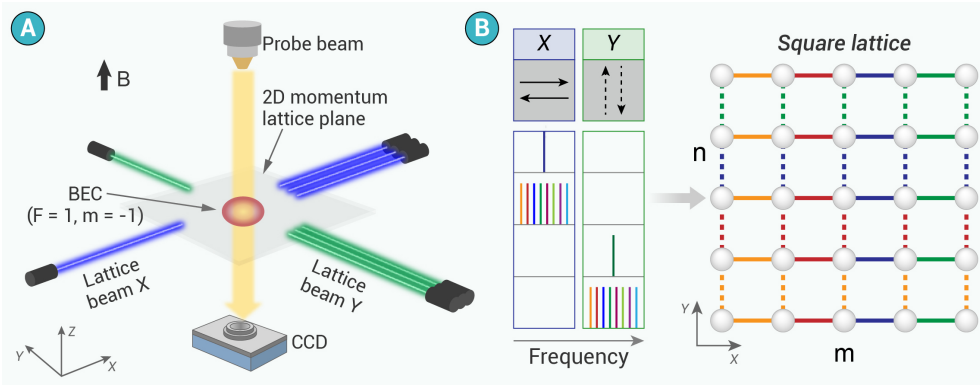


Figure 1. Schematic of constructing the 2D momentum lattice (A) Schematics of constructing the 2D momentum lattice. Two sets of lattice beams are applied along the x and y directions to couple the momentum states in both directions. (B) (Left) The radio frequency spectrum applied for generating the momentum lattice in the x and y directions. (Right) Illustration of the generated 2D square lattice, bonds with the same color and line shape share the same hopping rate, as they undergo the same two-photon process.

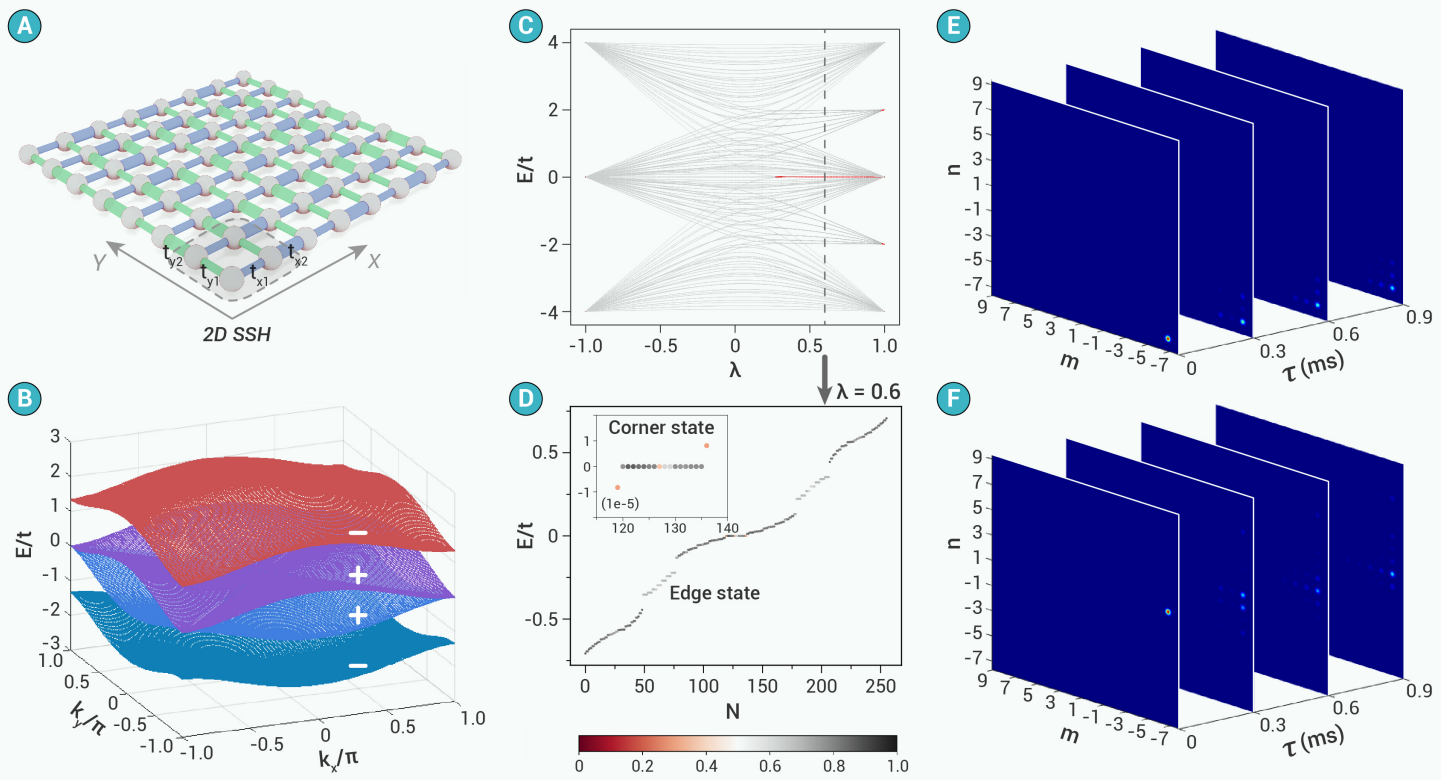


Figure 2. The dynamics of corner and edge bound-states (A) Illustration of the 2D SSH model, $t_{x1/y1}$ ($t_{x2/y2}$) are the intracell (intercell) hopping rates between adjacent sites along the x and y directions. (B) Bulk band spectrum in the higher-order topological phase with $\lambda = 0.5$. (C) The quasienergy spectrum of the 2D SSH model in various topological phases. The zero-dimensional corner states are shown in red in the topologically nontrivial region. This spectrum is calculated based on a finite square lattice under the open boundary condition. (D) The quasienergy spectrum of the 2D SSH model with $\lambda = 0.6$. There are edge states in the band gap. The zero-energy bulk states, that is, within a continuum. (E-F) Experimental results of the corner/edge-state dynamics up to $\tau = 0 \sim 0.9$ ms.

site (m, n) , and $t_{x1/y1} = t(1 - \lambda_{x/y})$ and $t_{x2/y2} = t(1 + \lambda_{x/y})$ are the intracell and intercell hopping rate between adjacent sites along the corresponding lattice direction, respectively. In our experiments, we primarily focus on the scenario with $\lambda_{x/y} = \lambda$ and denote $t_{x1/y1} = t_1$ and $t_{x2/y2} = t_2$, where $\lambda \in (-1, 1)$. For the parameterization $t_1 = t(1 - \lambda)$ and $t_2 = t(1 + \lambda)$, the regime $\lambda < 0$ corresponds to the topologically trivial phase, whereas $\lambda > 0$ corresponds to the topologically non-trivial phase.

The 2D SSH model, described by H_{eff} , permits two distinct topological phases that depend on the ratio of the intracell and intercell hopping rates.⁶³ Figure 2B displays the typical bulk energy spectrum of the topological nontrivial phase. Note that when we interchange the intracell and intercell hopping rates, both the topological trivial and nontrivial phases have the same bulk band structure. As shown in Figure 2C, the topological phase transition occurs at $\lambda = 0$, where the bulk band gap of Figure 2B closes at the high-symmetry points, a small residual gap remains due to the finite-size effect. The quasienergy spectrum under the open boundary condition features two branches of edge states as the system transits from the topologically trivial phase to the nontrivial region. In the topologically nontrivial phase, the model possesses a second-order topological phase, character-

ized by a 2D Zak phase⁶³ in the bulk, and zero-dimensional corner states at the boundary (marked in red in Figure 2C).

Particularly, we show the energy spectrum of a 16×16 square lattice for $\lambda = 0.6$ in Figure 2D. To characterize the localization of the eigenstates, we adopt the parameter $D_2 = -\ln(IPR)/\ln L$, where $IPR = \sum_j |\psi_j(j)|^4$ is the inverse participation ratio of eigenstate $\psi_j(j)$ with eigenenergy E_j , and L is the size of 2D lattice array. For a sufficiently large L , an eigenstate is considered extended when $D_2 \sim 1$, and localized when $D_2 \sim 0$. As labelled in the subfigure of Figure 2D, there are several zero-energy corner states embedded in the continuum of the zero-energy bulk states. These zero-dimensional corner states are the high-order topological BICs, protected by the bulk topological invariant of the 2D SSH model, which is itself a HOTI.

RESULTS

Observing bound-states dynamics

Notably, the corner states are protected by the C_{4v} and chiral symmetries, so that they do not hybridize with the bulk states. Although the corner BICs are spectrally degenerate with the zero-energy bulk states, hybridization is forbidden as long as the protecting symmetries are preserved. The chiral

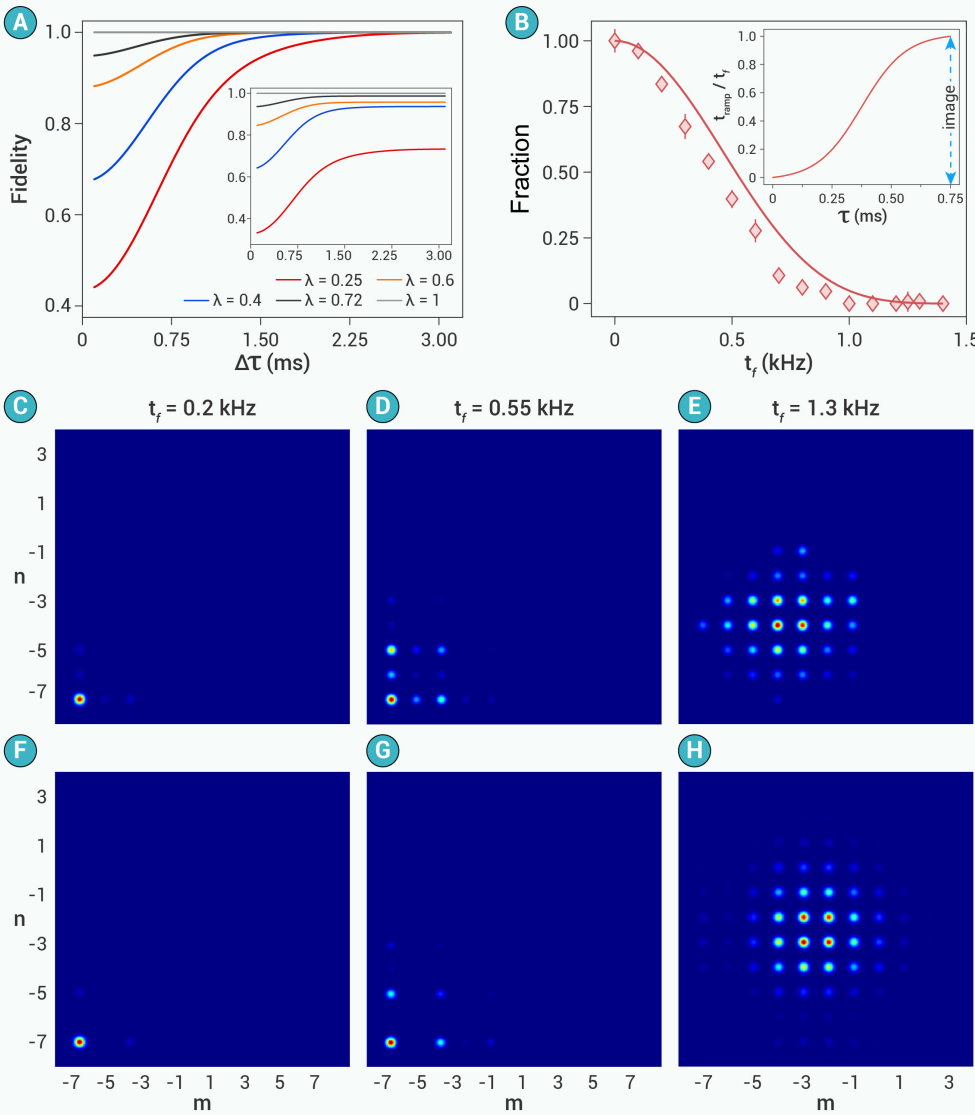


Figure 3. Adiabatic preparation of the higher-order BICs (A) The simulated fidelity of adiabatically preparing the target BICs for various ramping periods $\Delta\tau$ and different λ values in a 2D array of 16×16 . The inset shows the simulated fidelity of the 2D array of 15×15 . In both cases, t_2 is set to be $h \times 1.25$ kHz. (B) Variation of the residual condensate fraction in the initial site for different t_f values. The inset shows the variation of t_1 for the adiabatic ramping sequence. (C-E) The final states at the end of the adiabatic ramp, with $t_f = h \times 0.20(1)$ kHz, $0.55(2)$ kHz, and $1.30(2)$ kHz, respectively. (F-H) The numerically simulated results, with $\tau = 0.75$ ms, for $t_f = h \times 0.2$ kHz, 0.55 kHz, and 1.3 kHz, respectively.

rate. Although the adiabatic preparation of the BICs might seem counterintuitive due to the absence of an energy gap, the process is in fact protected by the C_4 symmetry, such that the bulk states nearby the BIC have vanishingly small overlap with the corner state and are not easily excited (see Supplemental Material).

Figure 3A presents the simulated fidelity of the adiabatic preparation, under various ramping periods $\Delta\tau$ and different λ values in a 2D 16×16 array. The inset of Figure 3A shows similar simulation results in a 2D 15×15 array. The simulated fidelity is defined as the overlap between the evolved state and the target zero-energy corner BIC. They both show that the validity of this adiabatic preparation can be guaranteed for relatively long ramping time. The main difference between $L = 16$ and $L = 15$ mainly comes from the broken C_4 symmetry in the latter case. In the case with an odd L , the target BICs, localized in one corner and exhibiting broken symmetry, cause the final fidelity to converge to a fixed value, which approaches unity as λ approaches 1. While in the case of even L , the BICs are distributed across four corners under the C_4 symmetry, and show perfect fidelity for long enough ramping times.

symmetry pins the corner modes to zero energy, while the C_{4v} symmetry and the lattice structure place the corner modes and the relevant bulk modes in orthogonal symmetry sectors. Therefore, the matrix elements coupling the corner BICs to the bulk continuum vanish in the ideal 2D SSH model. This gives rise to intrinsic corner bound-states dynamics that can be easily verified experimentally. To investigate this, we initialize the BEC at either a corner site or the midpoint of an edge, allowing us to examine the bound-states dynamics. Figure 2E shows the population at variable evolution times τ , when atoms are initialized at the corner site. Here the large overlap with the corresponding zero-energy BIC restricts the wave function diffusion into the bulk, leading to the observed corner-localized dynamics. Correspondingly, Figure 2F shows that the dynamics remains edge-localized when atoms are injected into the midpoint of the left edge, due to confinement of edge states along the 1D edge.

Symmetry-protected adiabatic preparation of BICs

Leveraging the programmability of the 2D momentum lattice, the higher-order topological BICs can be adiabatically prepared by implementing a symmetry-preserving time-dependent Hamiltonian. We begin by populating the corner site, which is exactly the zero-energy BICs in the limit of hopping rate, $t_1 = 0$ and $t_2 = t(1 + \lambda)$. Subsequently, we adiabatically ramp up t_1 from zero to t_f . The overall process is governed by the following time-dependent Hamiltonian

$$H(\tau) = -\sum_n (\sum_{m \in \text{odd}} t_{\text{ramp}}(\tau) c_{m+1,n}^\dagger c_{m,n} + \sum_{m \in \text{even}} t_2 c_{m+1,n}^\dagger c_{m,n}) - \sum_m (\sum_{n \in \text{odd}} t_{\text{ramp}}(\tau) c_{m,n+1}^\dagger c_{m,n} + \sum_{n \in \text{even}} t_2 c_{m,n+1}^\dagger c_{m,n}) + H.c. \quad (2)$$

where τ is the evolution time, $t_{\text{ramp}}(\tau)$ represents the time-dependent hopping

In our experiment, the initial BEC wave packet can only be prepared in a single corner of the 2D array. Consequently, the actual adiabatic preparation process closely resembles the odd- L case shown in Figure 3A. We fix $t_2 = h \times 1.25(2)$ kHz and adiabatically ramp up t_1 from zero to t_f over a duration of 0.75 ms, as shown in the inset of Figure 3B. Experimentally, we use the residual condensate fraction at the initially occupied corner cell as a directly measurable proxy for the localization of the prepared state. A large residual fraction indicates that the condensate remains localized near the corner, while a small value indicates diffusion into the bulk. Figures 3C-D depict the experimentally prepared BICs at $\tau = 0.75$ ms with $t_f = h \times 0.20(1)$ kHz and $0.55(2)$ kHz, respectively. Figures 3F-G show the corresponding numerically simulated results for the two cases. In either case, the final occupation is highly localized at even sites, consistent with the target zero-energy corner state. In contrast, when t_f becomes larger, as shown in Figures 3E & H, the final occupation diffuses into the bulk, indicating the breakdown of the adiabatic preparation. The gradual breakdown of the adiabaticity is reflected in Figure 3B, where we show this residual condensate fraction in the original corner site as a function of t_f .

Measuring higher-order topological phase transition

To demonstrate the higher-order topological nature of the BICs, we match the measurements of the bulk topological invariants and the corner states, through bulk and boundary dynamics, respectively.

Theoretically, the higher-order topological phase transition is captured by the 2D Zak phases.⁶³ In Figure 4A, we plot the topological phase diagram characterized by the Zak phases. Experimentally, we probe the topological phase transition along the diagonal of the phase diagram Figure 4A by

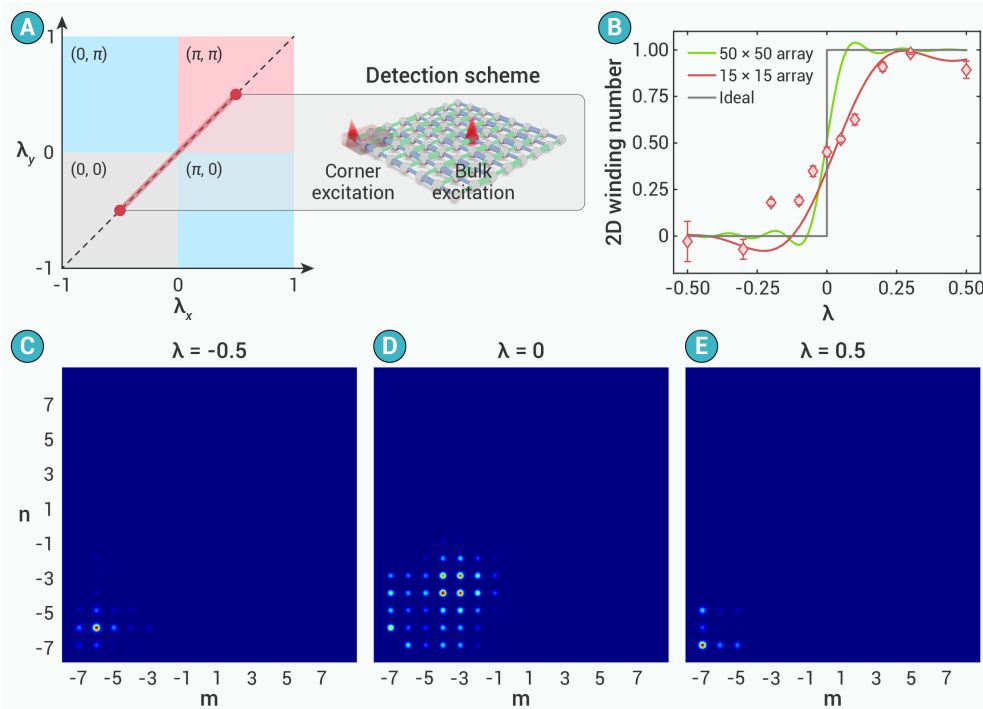


Figure 4. Measuring the higher-order topological phase transition. (A) The theoretical topological phase diagram with different λ_x and λ_y , characterized by the 2D Zak phase. The subfigure illustrates our measurements of the phase transition through the corner-site and bulk-site excitations, respectively. (B) The extracted 2D winding number varying with different λ . The dots are experimental data with $t = h \times 0.98(2)$ kHz, and the solid lines are numerical simulations using H_{eff} . (C-E) The experimental results following corner-site injection after 0.5 ms evolution with different λ (hence different topological phases).

manipulation of higher-order topological quantum matter in the strongly correlated regime.^{67,68}

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measuring the averaged 2D mean chiral displacement. More explicitly, we define the time-averaged mean chiral displacement as

$$v_{2d} = \frac{1}{\tau} \int_0^\tau \bar{P}_e(\tau) d\tau, \quad (3)$$

where $\bar{P}_e(\tau) = \langle \psi_0 | e^{iH_{\text{eff}}\tau} \mathcal{C} e^{-iH_{\text{eff}}\tau} | \psi_0 \rangle$, $\mathcal{C} = \hat{x}\Gamma_x + \hat{y}\Gamma_y$, with the chiral symmetry operators along the two spatial directions $\Gamma_x = \sigma_z \otimes I$, $\Gamma_y = I \otimes \sigma_z$. Here, σ_z is the Pauli matrix acting on the lattice degree of freedom, and I is the identity matrix, and τ is the total evolution time. For sufficiently long evolution time, v_{2d} oscillates around the 2D winding number extracted from the 2D Zak phases,⁶³ providing a bulk dynamic probe to the topological invariant.

For the experiment, we initialize the BEC at the central site of the 2D lattice, as indicated in the subfigure of Figure 4A. We then perform time evolutions with $\tau = 0.6$ ms, and calculate v_{2d} according to Eq. (3). Figure 4B summarizes a series of measurements with the lattice parameter λ varying from -0.5 to 0.5. Consistent with the theoretical prediction, we observed that in the topological trivial phase ($\lambda < 0$), the measured v_{2d} oscillates around 0, corresponding to the 2D Zak phases (0,0) of Figure 4A. While for the non-trivial phase ($\lambda > 0$), the measured v_{2d} oscillates around 1, corresponding to the 2D Zak phases (π, π) of Figure 4A.

To match the measurement results above, we further probe the BICs in different topological phases using boundary dynamics. We initialize the condensate at the corner site of the 2D array and measure the ensuing population evolution. In the topological trivial phase ($\lambda = -0.5$), Figure 4C shows that the measured population distribution at $\tau = 0.5$ ms already diffuses toward the bulk. While in the topological non-trivial phase ($\lambda = 0.5$), Figure 4E exhibits localized population, consistent with the emergence of the BIC. At the critical point ($\lambda = 0$), a fully diffusive population of the bulk state is observed, as depicted in Figure 4D.

DISCUSSION

In conclusion, we have experimentally created a 2D momentum lattice in an ultracold gas of ^{87}Rb atoms. Using this 2D programmable platform, we have demonstrated the design and manipulation of 2D SSH model, observing the characteristic dynamics of corner and edge bound states. Facilitated by the flexible control of our setup, we engineered a time-dependent Hamiltonian to adiabatically prepare the zero-energy BICs. We further harness the bound-state dynamics and the 2D topological invariant to measure the intrinsic higher-order topological phase transition.

For future studies, it would be worthwhile to extend our present 2D lattice configuration to more complicated geometries and internal-state configurations.⁶⁴ Additionally, the tuning of long-range interactions inherent to the momentum-lattice^{55,65-66} would also provide possibilities to study the

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AUTHOR CONTRIBUTIONS

H.W., H.L., and Z.D. contributed equally to this work. H.W., H.L., Z.D., and B.Y. conceived the project. H.W., Z.D., and Y.P. performed the experiments and analyzed the data. H.W., H.L., W.Y., and B.Y. contributed to the theoretical interpretation. H.W., H.L., W.Y., and B.Y. wrote the manuscript with input from all authors. H.L., W.Y., and B.Y. supervised the project. All authors contributed to the manuscript and approved the final version.

DECLARATION OF INTERESTS

The authors declare no competing interests.

ETHICAL STATEMENT AND PATIENT CONSENT

No human or animal subjects requiring ethical approval were involved in this study.

DATA AND MATERIALS AVAILABILITY

Data and code supporting the findings of this study are available from the corresponding authors upon reasonable request.

SUPPLEMENTAL INFORMATION

It can be found online at <https://doi.org/10.59717/j.tip.2026.100015>